Tutorial 13

Subspaces

In this course, we have seen many examples of vector spaces. One of the most common has been the x-y plane, a 2-dimensional vector space in which we have vectors of the type $\begin{pmatrix} a \\ b \end{pmatrix}$

In this example, $\begin{pmatrix} a \\ b \end{pmatrix}$ describes a point whose location can be reached by starting at the origin and travelling a distance a along the x-axis, followed by travelling a distance b parallel to the y-axis.

However, the vector $\begin{pmatrix} a \\ b \end{pmatrix}$ isn't limited to representing a physical position. Let's say that we have two functions of x. The first is x and the second is x^2. We can represent all the functions of the type $ax+bx^2$ with the vector $\begin{pmatrix} a \\ b \end{pmatrix}$. For example, we represent $3x+2x^2$ with the vector $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

The reason that this is useful is that addition and subtraction of the functions $ax+bx^2$ corresponds to addition and subtraction of their representative vectors.

For example, we represent $2x+x^2$ with $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $5x+2x^2$ with $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$

$2x+x^2+5x+2x^2 = 7x+3x^2$ which we represent with $\begin{pmatrix} 7 \\ 3 \end{pmatrix}$. $\begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} 7 \\ 3 \end{pmatrix}$.

So, with this notation, the first component represents the amount of x present in the function and the second component represents the amount of x^2 present.

We can extend this approach to any set of linearly independent functions. $ax+bx^2+cx^3$ could be represented with the 3-dimensional vector $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ and $a\sin x + b\cos x$ could be represented with the 2-dimensional vector $\begin{pmatrix} a \\ b \end{pmatrix}$.

We can also apply this idea to matrices. Let's look at all the matrices of the form $\begin{pmatrix} a & b \\ c & 0 \end{pmatrix}$, ie those where the (2,2) element is zero. In a 3 dimensional vector space, we can associate with the first axis, matrices of the form $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$. With the second and third axes, we associate matrices of the form $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ and $\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ respectively. So a general matrix $\begin{pmatrix} a & b \\ c & 0 \end{pmatrix}$, can be represented with a vector $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$.

Appropriately, addition of the vectors $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ corresponds to addition of the matrices $\begin{pmatrix} a & b \\ c & 0 \end{pmatrix}$.

Similarly, we can extend this idea to more complicated matrices.