top of the tetrahedron or near the bottom.

We start by taking the infinitely small square $dxdy$. The hypotenuse of the triangular slice is determined by the equation $x+\frac{y}{2}+\frac{z}{3}=1$, so we rearrange this to give our upper x limit $x=1-\frac{y}{2}-\frac{z}{3}$.

$$\text{Area of slice} = \int\int_0^{1-\frac{y}{2}-\frac{z}{3}} dxdy.$$

To work out the upper limit of the y-integration, we set $x=0$ in the equation $x+\frac{y}{2}+\frac{z}{3}=1$ and rearrange it to get y.

So the upper limit of y-integration is $y=2-\frac{2z}{3}$

$$\text{Area of slice} = \int_0^{2-\frac{2z}{3}}\int_0^{1-\frac{y}{2}-\frac{z}{3}} dxdy$$

$$\therefore \text{Volume of slice} = \int_0^{2-\frac{2z}{3}}\int_0^{1-\frac{y}{2}-\frac{z}{3}} dxdydz$$

$$\therefore \text{Mass of slice} = \rho\int_0^{2-\frac{2z}{3}}\int_0^{1-\frac{y}{2}-\frac{z}{3}} dxdydz$$

Now we must sum up all the slices. The lowest has z-coordinate 0 and the highest 3, so

$$\text{Mass of tetrahedron} = \rho\int_0^3\int_0^{2-\frac{2z}{3}}\int_0^{1-\frac{y}{2}-\frac{z}{3}} dxdydz$$

$$= \rho\int_0^3\int_0^{2-\frac{2z}{3}} \left[x\right]_0^{1-\frac{y}{2}-\frac{z}{3}} dydz = \rho\int_0^3\int_0^{2-\frac{2z}{3}} \left(1-\frac{y}{2}-\frac{z}{3}\right)dydz$$

$$= \rho\int_0^3\left[y-\frac{y^2}{4}-\frac{zy}{3}\right]_0^{2-\frac{2z}{3}} dz = \rho\int_0^3\left(2-\frac{2z}{3}-\frac{\left(2-\frac{2z}{3}\right)^2}{4}-\frac{2z-\frac{2z^2}{3}}{3}\right)dz$$

$$= \rho\int_0^3\left(2-\frac{2z}{3}-1+\frac{8z}{12}-\frac{4z^2}{36}-\frac{2z}{3}+\frac{2z^2}{9}\right)dz$$

$$= \rho\int_0^3\left(1-\frac{2z}{3}+\frac{z^2}{9}\right)dz = \rho\left[z-\frac{z^2}{3}+\frac{z^3}{27}\right]_0^3 = \rho(3-3+1) = \rho$$

Mass of tetrahedron $= \rho$.