gives us the volume of the slice

$$\text{volume of slice} = \int_0^{z^{1/2}}\int_0^{2\pi} r\,d\theta\,dr\,dz.$$

Finally, summing up the slices gives

$$\text{volume of shape} = \int_0^4\int_0^{z^{1/2}}\int_0^{2\pi} r\,d\theta\,dr\,dz$$

Multiplying by the density, ρ, gives the mass

$$M = \rho\int_0^4\int_0^{z^{1/2}}\int_0^{2\pi} r\,d\theta\,dr\,dz = \rho\int_0^4\int_0^{z^{1/2}} 2\pi r\,dr\,dz$$

$$= \rho\int_0^4 \left[\pi r^2\right]_0^{z^{1/2}} dz = \rho\int_0^4 \pi z\,dz = \rho\left[\tfrac{1}{2}\pi z^2\right]_0^4$$

$$= 8\pi\rho.$$

Because of the circular symmetry around the z-axis, it's clear that the x and y coordinates of the centre of gravity are zero. It's only the z-coordinate that we shall have to calculate.

$$\bar{z} = \frac{\sum_{\text{slices}} (\text{mass of each slice}) \times (\text{z-coordinate of each slice})}{\sum_{\text{slices}} (\text{Mass of each slice})}$$

We have already seen that the volume of a slice is

$$\text{volume of slice} = \int_0^{z^{1/2}}\int_0^{2\pi} r\,d\theta\,dr\,dz.$$

Multiplying this by the density and the z-coordinate of the slice gives

$$\rho\int_0^{z^{1/2}}\int_0^{2\pi} z r\,d\theta\,dr\,dz.$$

Summing this expression over all the slices will give us the numerator in $\bar{z}$

$$\rho\int_0^4\int_0^{z^{1/2}}\int_0^{2\pi} z r\,d\theta\,dr\,dz.$$

The denominator in $\bar{z}$ is just the mass of the whole shape and we have already concluded that this is $8\pi\rho$.

So $$\bar{z} = \frac{\rho\int_0^4\int_0^{z^{1/2}}\int_0^{2\pi} z r\,d\theta\,dr\,dz}{8\pi\rho} = \frac{1}{8\pi}\int_0^4\int_0^{z^{1/2}} z\,2\pi r\,dr\,dz$$