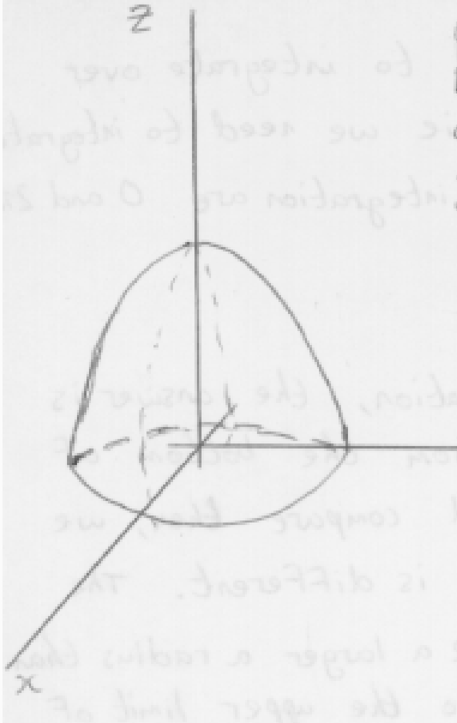

when $x=y=0$, $z=4$ so the height is 4.
Because the shape is symmetric across the y-z plane and is symmetric about the x-z plane, the x and y coordinates of the centre of mass must both be zero. This leaves us just the z-coordinate to calculate.

Taking thin slices again, the area of each slice is $\iint r\,d\theta\,dr$. The limits on the θ-integration must be 0 and 2π, but to get the limits for the r-integration, we must look at the equation $z=4-x^2-y^2$. Rearranging it into the form of the equation of a circle gives $x^2+y^2=4-z$. Comparing this to $x^2+y^2=r^2$ tells us that $r^2=4-z$ or $r=\sqrt{4-z}$. This is our upper limit on the r-integration.

Area of each slice $= \displaystyle\int_0^{\sqrt{4-z}}\int_0^{2\pi} r\,d\theta\,dr.$

Multiplying by the thickness of the slice, dz, and summing over the slices gives the volume as $\displaystyle\int_0^4\int_0^{\sqrt{4-z}}\int_0^{2\pi} r\,d\theta\,dr\,dz.$

Because the shape has constant density, its mass is $\displaystyle\rho\int_0^4\int_0^{\sqrt{4-z}}\int_0^{2\pi} r\,d\theta\,dr\,dz$, where ρ is the density.

In tutorial 10, we saw that the centre of mass coordinate was defined as $\dfrac{\sum(\text{mass of each chunk})\times(\text{position of each chunk})}{\sum(\text{mass of each chunk})}$.

$\displaystyle\rho\int_0^4\int_0^{\sqrt{4-z}}\int_0^{2\pi} r\,d\theta\,dr\,dz$ is our denominator for this expression.

For the numerator, we need to take the mass of each slice, $\displaystyle\rho\int_0^{\sqrt{4-z}}\int_0^{2\pi} r\,d\theta\,dr\,dz$, multiply it by its position to get $\displaystyle\rho\int_0^{\sqrt{4-z}}\int_0^{2\pi} z\,r\,d\theta\,dr\,dz$ and then sum over all the slices

$$\rho\int_0^4\int_0^{\sqrt{4-z}}\int_0^{2\pi} z\,r\,d\theta\,dr\,dz.$$