closer to parallel. In the extreme, where δ is infinitely small, the two sides can be assumed to be parallel and the shaded region becomes an infinitely thin rectangle.

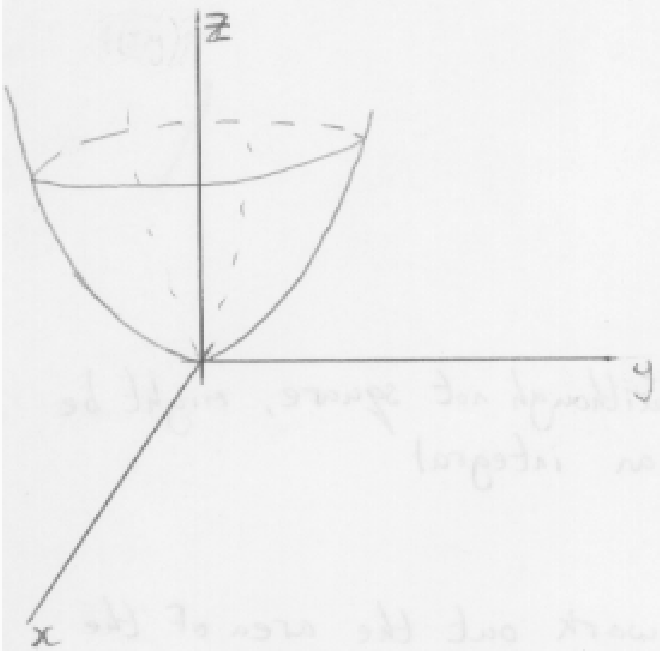
IF we make s infinitely small, too, then the shaded region becomes an infinitely small square. To reflect this, we rename s as dr and δ as $d\theta$. The area of the region becomes $r\,d\theta\,dr$.

Using integral signs to denote the sum over the infinitely small squares, the area in polar coordinats is

$$\text{Area} = \iint r\,d\theta\,dr.$$

The limits in the inner integral are the range of the values of θ and in the outer integral, the range of values of r.

Now, let's have a look at volumes. There are different ways to calculate the volume. Here we shall only look at cylindrical polar coordinates, in which we use polar coordinates and the conventional z-coordinate as our integration coordinates.



Suppose we have a 'bowl' shaped region whose volume we want to calculate. The bowl has height z_a and its sides are determined by the equation $z = x^2 + y^2$

One way to calculate the volume would be to cut the bowl into thin, circular slices. Using polar coordinates, we could work out the area of each slice, multiply the area by the thickness of each slice, to get the volume, and then add up the volumes of all the slices.

Using the formula from above, the area of a circular slice is $\iint r\,d\theta\,dr$. IF each slice is infinitely thin, then the volume will be $\iint r\,d\theta\,dr\,dz$ and if we add up all the slices, we will have $\iiint r\,d\theta\,dr\,dz$ = volume.