Summing over all the strips gives us $\text{Area} = \sum \frac{1}{2} r^2(\theta)\,\alpha$. The limits on the sum are $\theta = -\frac{\pi}{4}$ and $\frac{\pi}{4}$ because these are the angles at which the curve become imaginary (ie the square root of a negative number).

So $\text{Area} = \sum_{\theta=-\frac{\pi}{4}}^{\pi/4} \frac{1}{2} r^2(\theta)\,\alpha$.

As α becomes infinitely small, this becomes an exact expression. In this extreme, we replace α with $d\theta$ and $\sum_{\theta=-\frac{\pi}{4}}^{\pi/4}$ with $\int_{-\frac{\pi}{4}}^{\pi/4}$

$$\text{Area} = \int_{-\frac{\pi}{4}}^{\pi/4} \frac{1}{2} r^2(\theta)\, d\theta$$

substituting for r with $\sqrt{\cos 2\theta}$, we get

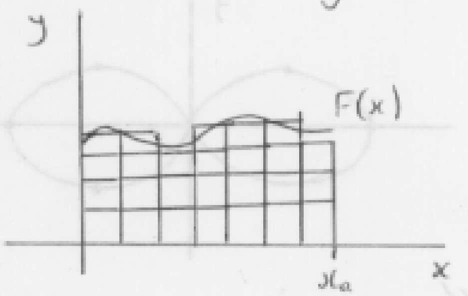
$$\text{Area} = \int_{-\pi/4}^{\pi/4} \frac{1}{2}\cos 2\theta\, d\theta.$$

Evaluating this gives $\text{Area} = \left[\frac{1}{4}\sin 2\theta\right]_{-\pi/4}^{\pi/4} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}.$

Before we go on to question 2, we need to look at a bit more theory.

So far, we have looked at ways to find the area of well defined regions. First, we will look at an alternative way to do this and then we will extend the idea to working out the volume.

Suppose that, instead of dividing regions up into strips, we divided them up into squares. We could approximate the area under a curve by summing up the areas of each square.



If the width of each square is s and the height t and each square is centred at a point (x,y)

$$\text{Area} = \sum_{x=0}^{x_a} \sum_{y=0}^{y_{max}(x)} s\,t$$

The first sum adds up a column of squares. However, because the tops of the columns of squares follows the curve $F(x)$, the columns don't all have the same height. In fact, the height of the columns depends on the x-coordinate, so the y-summation finishes at $y_{max}(x)$.

Making the width, s, and the height, t, of each square infinitely small we get an exact expression for the area under the curve. $\text{Area} = \int_0^{x_a}\int_0^{F(x)} dy\,dx$