Tutorial 11

Question 1

Plot the curve in polar coordinates $r = +\sqrt{\cos 2\theta}$ and find the area of one loop.

Solution

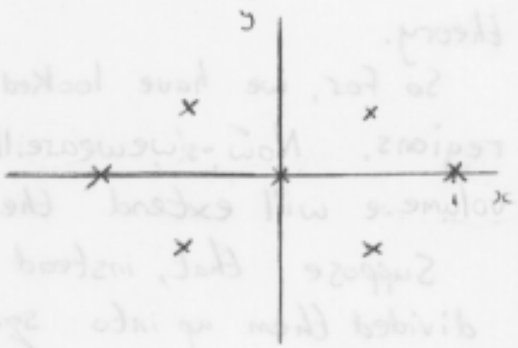
In order to plot the curve, we need to work out r for a range of values of θ.

θ	r
0	1
$\pi/8$	$\sqrt{\frac{1}{2}}$
$\pi/4$	0
$3\pi/8$	$\sqrt{\frac{-1}{2}}$
$\pi/2$	$\sqrt{-1}$
$5\pi/8$	$\sqrt{\frac{-1}{2}}$
$3\pi/4$	0
$7\pi/8$	$\sqrt{\frac{1}{2}}$
π	1

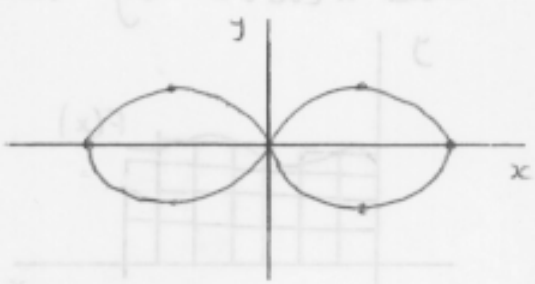
θ	r
$9\pi/8$	$\sqrt{\frac{1}{2}}$
$5\pi/4$	0
$11\pi/8$	$\sqrt{\frac{-1}{2}}$
$3\pi/2$	$\sqrt{-1}$
$13\pi/8$	$\sqrt{\frac{-1}{2}}$
$7\pi/4$	0
$15\pi/8$	$\sqrt{\frac{1}{2}}$
2π	1

We are only interested in the real values of r, so where r is the square root of a negative number (ie where it is imaginary) we don't plot it.

If we plot these points, we get (we have ignored the points that involve the square root of a negative number).

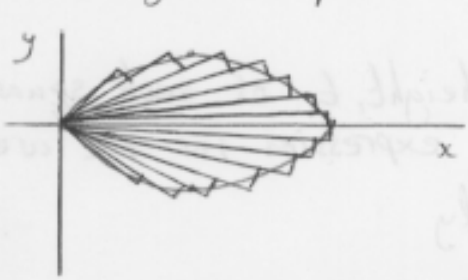


Joining the points with a curve gives us this for the plot.



When the question asks for the area of one loop, it means one of the two loops that lie on either side of the y-axis. It doesn't matter which loop we pick. We shall calculate the area of the loop on the right of the y-axis.

As was explained in tutorial 10, we start by dividing the loop up into triangular strips.



The area of each strip is $\frac{1}{2}$(base)(height)
$= \frac{1}{2}(r)(r\alpha)$ where α is the angle between the two longest sides of each triangular strip and r, the length of each strip, depends on the angle between the strip and the x-axis