

$$\text{So } \bar{y} = \frac{\int_0^\pi y \rho ds}{\int_0^\pi \rho ds}$$

Substituting $\pi a \rho$ For $\int_0^\pi \rho ds$ and $a d\theta$ For ds gives us

$$\bar{y} = \frac{\int_0^\pi y \rho a d\theta}{\pi a \rho}$$

but y is a Function of θ . $y = a \sin \theta$
so we can substitute For it.

$$\bar{y} = \frac{\int_0^\pi \rho a a \sin \theta d\theta}{\pi a \rho} = \frac{\int_0^\pi \rho a^2 \sin \theta d\theta}{\pi a \rho}$$

$$\text{evaluating this, } \bar{y} = \frac{[\rho a^2 (-\cos \theta)]_0^\pi}{\pi a \rho} = \frac{2 \rho a^2}{\pi a \rho} = \frac{2a}{\pi}$$

So the centre of mass lies at $(0, \frac{2a}{\pi})$, in x and y coordinates.