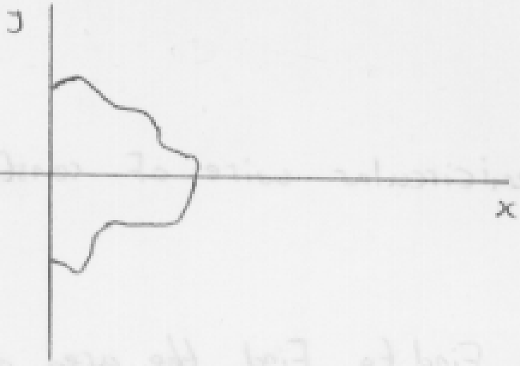
In effect, θ tells us what direction, from the origin, the point lies in and r tells us how far from the origin it lies.

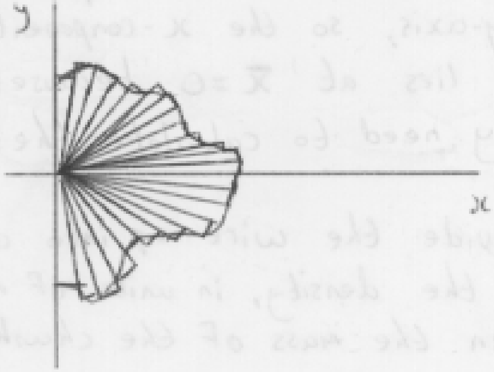
Suppose that we have a function that we want to integrate, using polar coordinates. The function might look something like



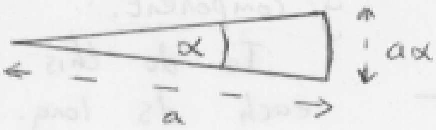
and we want to find the area enclosed by the function and the y-axis.

In terms of the r and θ variables, the function must give r in terms of θ because in every direction θ, the distance from the origin to the point on the curve, depends on the function.

We follow a similar approach to the x-y case and divide the shape up into strips. However, the strips are now triangular.



With finite width, adding up the area of all the strips provides us with an approximation to the area of the shape.



The area of each triangular strip is $\frac{1}{2}$(base)(height)

which is $\frac{1}{2}(a)(a\alpha)$. The reason that the height is $a\alpha$, becomes clear if we think of the circumference of a circle. The circumference, C, $= 2\pi r$, where r is the radius of the circle and 2π is the angle through which you pass, when you go right the way round the circle. So the length of an arc that subtends an angle β is $r\beta$. In the case above, the radius is a and the angle is α, so the length of the arc is $a\alpha$. Strictly speaking, the height of the triangle is not the same as the length of the arc. However, later on we shall restrict ourselves to keeping α small and the smaller α gets the closer the arc length gets to the triangle's height.

So summing up over all the strips, where the base of the strip, a, is a function of the angle, we have

$$\sum \tfrac{1}{2}(a(\theta))(a(\theta)\alpha) = \sum \tfrac{1}{2}(a(\theta))^2\alpha$$

In the extreme, where we make α infinitely small, this equation ceases to be an approximation and becomes exact. Changing our notation to reflect this, we replace α with $d\theta$ and Σ with $\int$ to give