we can use the symmetry of the region to our advantage. Because the mass distribution of the region is the same on either side of the point $x=0$, the x-component of the centre of mass must be $\bar{x}=0$. This means that we only have to worry about the y-component.

Our strategy will be to have strips running in the x-direction, from the y-axis, at one end, to the point $x=+\sqrt{1-y^2}$ at the other end. By doing this we are effectively ignoring the left hand half of the shape. But if we multiply the area of each strip by a factor of 2, it will compensate for omitting the other half of the shape.

So the mass of each strip is $2\rho\sqrt{1-y^2}\,dy$. Multiplying this by y and summating gives $\int_0^1 2\rho y\sqrt{1-y^2}\,dy$ as the numerator in the expression for $\bar{y}$. For the denominator, we can just use the standard expression for the area of half a circle, $\frac{1}{2}\pi r^2$, and multiply it by the density.

This gives $\bar{y} = \dfrac{\int_0^1 2\rho y\sqrt{1-y^2}\,dy}{\frac{1}{2}\pi\rho} = \dfrac{4}{\pi}\int_0^1 y\sqrt{1-y^2}\,dy.$

using the substitution $u=y^2$, $\frac{du}{dy}=2y$ $\quad dy=\frac{du}{2y}$

At $y=0$, $u=0$ and at $y=1$, $u=1$,

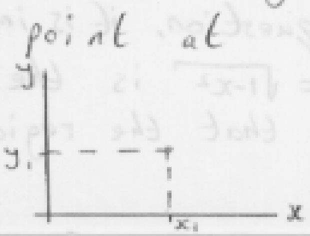
so $\bar{y} = \frac{4}{\pi}\int_0^1 y\sqrt{1-u}\,\frac{du}{2y} = \frac{4}{\pi}\int_0^1 \frac{1}{2}(1-u)^{1/2}\,du = \frac{4}{\pi}\left[-\frac{1}{3}(1-u)^{3/2}\right]_0^1$

$\bar{y} = \frac{4}{\pi}\left(\frac{1}{3}\right) = \frac{4}{3\pi}$

Before we proceed to question 2, we must first have a look at polar coordinates and how we integrate, using them.

(x,y) are known as Euclidean coordinates and we use them to identify any point on the x-y plane. An alternative way of doing this is to use polar coordinates in which every point on the x-y plane can be represented using the distance, r, from the origin and the angle θ to the x-axis

So for a point at position (x_1, y_1)



we can also describe it as being at position (r_1, θ_1)