So $\bar{x} = \dfrac{\int_0^3 \rho(3-x)x\,dx}{\int_0^3 \rho(3-x)\,dx} = \dfrac{\int_0^3 (3x-x^2)\,dx}{\int_0^3 (3-x)\,dx} = \dfrac{\left[\frac{3}{2}x^2 - \frac{1}{3}x^3\right]_0^3}{\left[3x - \frac{1}{2}x^2\right]_0^3}$

$$\bar{x} = \frac{\frac{27}{2} - 9}{9 - \frac{9}{2}} = \frac{\frac{9}{2}}{\frac{9}{2}} = 1$$

Now we need to work out the y-coordinate of the centre of mass. We do this by having the length of the strips run in the x-direction and setting the width of the strips to be dy.

We use $x+y=3$ again to get the length of each strip. However, this time we need to rearrange it to give x in terms of y. $x = 3-y$. So the area of the strip is $(3-y)dy$. Its mass is $\rho(3-y)dy$ and the numerator of $\bar{y}$ looks like $\int_0^3 \rho(3-y)y\,dy$. The denominator is the sum of the masses of each strip and looks like $\int_0^3 \rho(3-y)\,dy$.

So $\bar{y} = \dfrac{\int_0^3 \rho(3-y)y\,dy}{\int_0^3 \rho(3-y)\,dy} = \dfrac{\int_0^3 (3y-y^2)\,dy}{\int_0^3 (3-y)\,dy} = \dfrac{\left[\frac{3}{2}y^2 - \frac{1}{3}y^3\right]_0^3}{\left[3y - \frac{1}{2}y^2\right]_0^3} = \dfrac{\frac{27}{2} - 9}{9 - \frac{9}{2}}$

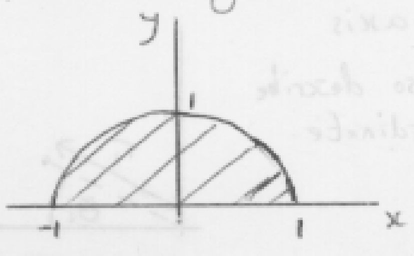
$$\bar{y} = \frac{\frac{9}{2}}{\frac{9}{2}} = 1$$

The centre of mass lies at the point $(1,1)$

To make sure that we have the right answer, it is a good idea to check that the point $(1,1)$ lies within the boundary of the region. From the diagram, we can see that it does.

With normal shapes, like our triangle, the centre of mass should always fall within the boundary of the shape. However, with hollow or concave shapes this may not be the case. For example, the centre of mass of a horse-shoe does not lie at a point on or within the surface of the horse-shoe.

b) The region bound by the x-axis and $y = \sqrt{1-x^2}$ looks like



To answer this question, it is important to realize that $y = \sqrt{1-x^2}$ is the equation of a circle and that the region is a half circle.