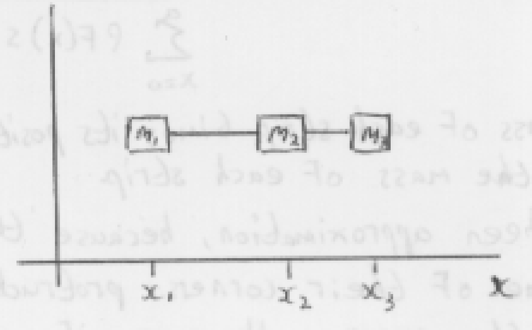
Tutorial 10 - Calculating the centre of mass



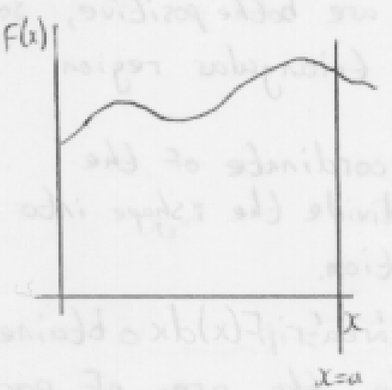
If we had three blocks, of mass M_1, M_2 and M_3, at positions x_1, x_2 and x_3, respectively, we would calculate their centre of mass with the expression.

$$\overline{x} = \frac{M_1 x_1 + M_2 x_2 + M_3 x_3}{(M_1 + M_2 + M_3)}$$

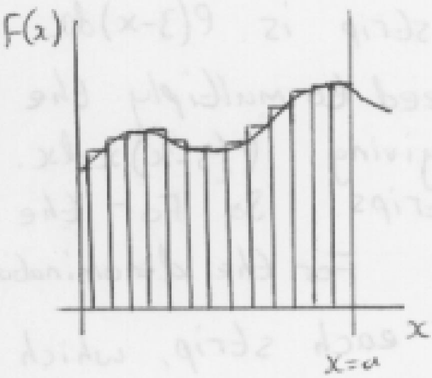
The point $\overline{x}$ is the place where the torque (which is the same as the moment) is balanced, on either side. So, if we held the rod, in the diagram above, at the point $\overline{x}$, it would balance.

By way of warning, the phrases centre of mass and centre of gravity are often used interchangeably. In the context of problems like the one above, they mean the same thing. They come to mean different things only when the gravitational field is not constant, but a function of position. This won't be a problem to us, because when we need to include a gravitational field, it will invariably be constant.

Now, the subject of this tutorial will be calculating the centre of mass of geometric shapes, using the integral constructions explained in the previous tutorial.



In the example on the left, we want to find the x-coordinate of the centre of mass of the shape bound by the x-axis, the lines $x=0$ and $x=a$ and the curve $F(x)$. We do this by, firstly, dividing the region up into strips.



The height of a strip centred at position x, is $F(x)$ and the width is s, so the area of the strip is $F(x)s$. The mass of the strip is $\rho F(x)s$, where ρ is the density defined in units of mass per unit area.

$\rho F(x)s$ is analagous to the M used in the example above

To work out $\overline{x}$ we need to multiply the mass of each strip by its position, sum over all the strips and then divide by the total mass of the shape. This is what we did above.