So we have two extreme points $\left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ and $\left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$

At $\left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ $\quad F = \frac{3}{\sqrt{3}} = \sqrt{3}$.

At $\left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$ $\quad F = \frac{-3}{\sqrt{3}} = -\sqrt{3}$.

4) a) $x^2 - y - 5z = 0$ at the point $(2,-1,1)$

$F = x^2 - y - 5z$

Normal vector $= \frac{\partial F}{\partial x}\underset{\sim}{i} + \frac{\partial F}{\partial y}\underset{\sim}{j} + \frac{\partial F}{\partial z}\underset{\sim}{k}$

$= 2x\underset{\sim}{i} - \underset{\sim}{j} - 5\underset{\sim}{k}$.

At $(2,-1,1)$, the Normal vector $= 4\underset{\sim}{i} - \underset{\sim}{j} - 5\underset{\sim}{k}$

As in question 5 of Homework I, we construct an arbitrary vector about $(2,-1,1)$ and set it to be perpendicular to the normal

$$\begin{pmatrix} 4 \\ -1 \\ -5 \end{pmatrix} \cdot \begin{pmatrix} x-2 \\ y+1 \\ z-1 \end{pmatrix} = 0$$

$4(x-2) - (y+1) - 5(z-1) = 0$

$4x - 8 - y - 1 - 5z + 5 = 0$

$4x - y - 5z - 4 = 0$ $\leftarrow$ Equation of the tangent plane.

b) $x^2 + y^2 + z = 4$ at the point $(1,1,2)$

$F = x^2 + y^2 + z - 4$

$\frac{\partial F}{\partial x} = 2x$. At $(1,1,2)$, $\frac{\partial F}{\partial x} = 2$

$\frac{\partial F}{\partial y} = 2y$. At $(1,1,2)$, $\frac{\partial F}{\partial y} = 2$

$\frac{\partial F}{\partial z} = 1$

Normal vector $= 2\underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}$

To calculate tangent plane,

$$\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} x-1 \\ y-1 \\ z-2 \end{pmatrix} = 0$$

$2(x-1) + 2(y-1) + (z-2) = 0$

$2x - 2 + 2y - 2 + z - 2 = 0$

$2x + 2y + z - 6 = 0$ $\leftarrow$ Equation of tangent plane.