So now we have 3 equations and 3 unknowns

① $3x^2 = 2x\lambda$

② $2y = 2y\lambda$

③ $x^2+y^2 = 1$

From ②, $\lambda = 1$. substituting For λ in ①, $3x^2 - 2x = 0$

$x(3x-2) = 0$

So $x = 0$ or $x = \frac{2}{3}$

$x^2+y^2 = 1$, $y^2 = 1-x^2$, $y = \pm\sqrt{1-x^2}$

IF $x = 0$, $y = \pm 1$

IF $x = \frac{2}{3}$, $y = \pm\sqrt{1-\frac{4}{9}} = \pm\sqrt{5/9} = \pm\frac{\sqrt{5}}{3}$

This gives us 4 points: $(0,1)$, $(0,-1)$, $\left(\frac{2}{3}, \frac{\sqrt{5}}{3}\right)$, $\left(\frac{2}{3}, -\frac{\sqrt{5}}{3}\right)$

The question asks For the extreme values oF F, so we must calculate F at these 4 points.

At $(0,1)$, $F = 1$

At $(0,-1)$, $F = 1$

At $\left(\frac{2}{3}, \frac{\sqrt{5}}{3}\right)$, $F = \frac{8}{27} + \frac{5}{9} = \frac{8}{27} + \frac{15}{27} = \frac{23}{27}$

At $\left(\frac{2}{3}, -\frac{\sqrt{5}}{3}\right)$, $F = \frac{8}{27} + \frac{5}{9} = \frac{8}{27} + \frac{15}{27} = \frac{23}{27}$.

B) $F = x-y+z$, $g = x^2+y^2+z^2-1$

$\nabla F = \lambda \nabla g$ $\quad \frac{\partial F}{\partial x} = 1$, $\frac{\partial F}{\partial y} = -1$, $\frac{\partial F}{\partial z} = 1$, $\frac{\partial g}{\partial x} = 2x$, $\frac{\partial g}{\partial y} = 2y$, $\frac{\partial g}{\partial z} = 2z$

$\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k} = \lambda(2x\underset{\sim}{i} + 2y\underset{\sim}{j} + 2z\underset{\sim}{k})$

Equating $\underset{\sim}{i}, \underset{\sim}{j}, \underset{\sim}{k}$ gives

① $1 = 2x\lambda$

② $-1 = 2y\lambda$

③ $1 = 2z\lambda$

and we have $x^2+y^2+z^2 = 1$

Rearranging ①, $\lambda = \frac{1}{2x}$. substituting For λ in ② & ③

$-1 = \frac{2y}{2x} = \frac{y}{x}$

$1 = \frac{2z}{2x} = \frac{z}{x}$

Now we have 3 equations $-1 = \frac{y}{x}$, $1 = \frac{z}{x}$, $x^2+y^2+z^2 = 1$

using the First we get $y = -x$ and the second gives $z = x$.

substituting For y and z in $x^2+y^2+z^2 = 1$ gives

$3x^2 = 1$, $x^2 = \frac{1}{3}$, $x = \pm\frac{1}{\sqrt{3}}$

IF $y = -x$, then when $x = \frac{1}{\sqrt{3}}$, $y = \frac{-1}{\sqrt{3}}$ when $x = -\frac{1}{\sqrt{3}}$, $y = \frac{1}{\sqrt{3}}$

IF $z = x$, then when $x = \frac{1}{\sqrt{3}}$, $z = \frac{1}{\sqrt{3}}$. when $x = -\frac{1}{\sqrt{3}}$, $z = -\frac{1}{\sqrt{3}}$