At $(0,0)$, $144xy - 9 < 0$ so $(0,0)$ is a saddle point

At $(-\frac{1}{2}, -\frac{1}{2})$ $144xy - 9 > 0$ so $(-\frac{1}{2}, -\frac{1}{2})$ is either a maximum or a minimum.

which?

$\frac{\partial^2 F}{\partial x^2} = 12x$. At $(-\frac{1}{2}, -\frac{1}{2})$ $\frac{\partial^2 F}{\partial x^2} < 0$ so $(-\frac{1}{2}, -\frac{1}{2})$ is a maximum.

c) $F = x^3 + y^3 + 3x^2 - 3y^2$. $\frac{\partial F}{\partial x} = 3x^2 + 6x$, $\frac{\partial F}{\partial y} = 3y^2 - 6y$.

① $3x^2 + 6x = 0$

② $3y^2 - 6y = 0$

Solving ① $x(3x+6) = 0$ $x = 0$ or $x = -2$

Solving ② $y(3y-6) = 0$ $y = 0$ or $y = 2$.

This gives 4 possible solutions, $(0,0), (0,2), (-2,0), (-2,2)$

$\frac{\partial^2 F}{\partial x^2} = 6x + 6$, $\frac{\partial^2 F}{\partial y^2} = 6y - 6$, $\frac{\partial^2 F}{\partial x \partial y} = 0$

$\left(\frac{\partial^2 F}{\partial x^2}\right)\left(\frac{\partial^2 F}{\partial y^2}\right) - \left(\frac{\partial^2 F}{\partial x \partial y}\right)^2 = (6x+6)(6y-6)$

At $(0,0)$, $\left(\frac{\partial^2 F}{\partial x^2}\right)\left(\frac{\partial^2 F}{\partial y^2}\right) - \left(\frac{\partial^2 F}{\partial x \partial y}\right)^2 < 0$, so $(0,0)$ is a saddle point.

At $(-2,2)$, $\left(\frac{\partial^2 F}{\partial x^2}\right)\left(\frac{\partial^2 F}{\partial y^2}\right) - \left(\frac{\partial^2 F}{\partial x \partial y}\right)^2 < 0$, so $(-2,2)$ is a saddle point

At $(0,2)$, $\left(\frac{\partial^2 F}{\partial x^2}\right)\left(\frac{\partial^2 F}{\partial y^2}\right) - \left(\frac{\partial^2 F}{\partial x \partial y}\right) > 0$, so $(0,2)$ is either a maximum or a minimum.

$\frac{\partial^2 F}{\partial x^2} > 0$ so $(0,2)$ is a minimum.

At $(-2,0)$, $\left(\frac{\partial^2 F}{\partial x^2}\right)\left(\frac{\partial^2 F}{\partial y^2}\right) - \left(\frac{\partial^2 F}{\partial x \partial y}\right) > 0$ so $(-2,0)$ is either a maximum or a minimum.

$\frac{\partial^2 F}{\partial x^2} < 0$ so $(-2,0)$ is a maximum.

3) a) $F = x^3 + y^2$ $x^2 + y^2 = 1$. We set $g = x^2 + y^2 - 1$

we need to solve $\nabla F = \lambda \nabla g$ and $x^2 + y^2 = 1$

$\nabla F = 3x^2 \underline{i} + 2y \underline{j}$ $\nabla g = 2x \underline{i} + 2y \underline{j}$

$\nabla F = \lambda \nabla g$ gives $3x^2 \underline{i} + 2y \underline{j} = \lambda(2x \underline{i} + 2y \underline{j})$

Equating the $\underline{i}$ and $\underline{j}$ components, $3x^2 = 2x\lambda$
$2y = 2y\lambda$