Rearranging ① $x = \frac{y-2}{2}$. If $y=-2$, then $x=-2$, which gives us the point $(-2,-2)$

$$\frac{\partial^2 F}{\partial x^2} = 2, \quad \frac{\partial^2 F}{\partial y^2} = 2, \quad \frac{\partial^2 F}{\partial x \partial y} = -1$$

so $\left(\frac{\partial^2 F}{\partial x^2}\right)\left(\frac{\partial^2 F}{\partial y^2}\right) - \left(\frac{\partial^2 F}{\partial x \partial y}\right)^2 = (2)(2) - (1) = 3.$

Because this is positive, ~~there are no saddle point~~ $(-2,-2)$ cannot be a saddle point.

So is $(-2,-2)$ a maximum or a minimum?

$\frac{\partial^2 F}{\partial x^2} = 2$, which is greater than zero. Therefore, $(-2,-2)$ must be a minimum.

b) $F = 2x^3 + 3xy + 2y^3$ $\quad \frac{\partial F}{\partial x} = 6x^2 + 3y, \quad \frac{\partial F}{\partial y} = 3x + 6y^2$

① $6x^2 + 3y = 0$
② $3x + 6y^2 = 0$

Rearranging ① gives $y = -2x^2$ and substituting for y in ② gives

$$3x + 6(-2x^2)^2 = 0$$
$$3x + 6(4x^4) = 0$$
$$3x + 24x^4 = 0$$
$$x(3 + 24x^3) = 0$$

Either $x=0$ or $3 + 24x^3 = 0$. If $3+24x^3=0$

$$1 + 8x^3 = 0$$
$$8x^3 = -1$$
$$x^3 = -1/8$$
$$x = -1/2$$

Either $x=0$ or $x = -1/2$

$y = -2x^2$. If $x=0$, then $y=0$

If $x=-1/2$, then $y = -1/2$

Our two stationary points are $(0,0)$ and $(-\frac{1}{2}, -\frac{1}{2})$. Are they maxima, minima or saddle points.

$$\frac{\partial^2 F}{\partial x^2} = 12x, \quad \frac{\partial^2 F}{\partial y^2} = 12y, \quad \frac{\partial^2 F}{\partial x \partial y} = 3$$

$$\left(\frac{\partial^2 F}{\partial x^2}\right)\left(\frac{\partial^2 F}{\partial y^2}\right) - \left(\frac{\partial^2 F}{\partial x \partial y}\right)^2 = 144xy - 9$$