Homework II

1) a) $F = r\cos\theta + r\sin\theta$

$$\frac{\partial F}{\partial r} = \cos\theta \frac{\partial r}{\partial r} + \sin\theta \frac{\partial r}{\partial r} = \cos\theta + \sin\theta.$$

$$\frac{\partial F}{\partial \theta} = r\frac{\partial}{\partial\theta}\cos\theta + r\frac{\partial}{\partial\theta}\sin\theta$$

To continue, we need to know that $\frac{\partial}{\partial\theta}\cos\theta = -\sin\theta$ and $\frac{\partial}{\partial\theta}\sin\theta = \cos\theta$.

So $\frac{\partial F}{\partial\theta} = -r\sin\theta + r\cos\theta$

b) $F = (R_1)^{-1} + (R_2)^{-1} + (R_3)^{-1}$

$$\frac{\partial F}{\partial R_1} = -(R_1)^{-2}, \quad \frac{\partial F}{\partial R_2} = -(R_2)^{-2}, \quad \frac{\partial F}{\partial R_3} = -(R_3)^{-2}$$

c) $F = nRTV^{-1}$

$$\frac{\partial F}{\partial n} = RTV^{-1}, \quad \frac{\partial F}{\partial R} = nTV^{-1}, \quad \frac{\partial F}{\partial T} = nRV^{-1}, \quad \frac{\partial F}{\partial V} = -nRTV^{-2}$$

2) At a saddle point, a maximum or a minimum, $\frac{\partial F}{\partial x} = 0$ & $\frac{\partial F}{\partial y} = 0$.

So we need to find the points where $\frac{\partial F}{\partial x} = 0$ & $\frac{\partial F}{\partial y} = 0$

Once we have worked out those points, we need to work out $\left(\frac{\partial^2 F}{\partial x^2}\right)\left(\frac{\partial^2 F}{\partial y^2}\right) - \left(\frac{\partial^2 F}{\partial x \partial y}\right)^2$ at each. If the answer is negative, then, at that point, F is a saddle point. Otherwise, F is either a maximum or a minimum.

$\frac{\partial^2 F}{\partial x^2}$ will tell us whether it is a maximum or a minimum.

If $\frac{\partial^2 F}{\partial x^2} < 0$, then it is a maximum. If $\frac{\partial^2 F}{\partial x^2} > 0$, then it is a minimum.

a) $F = x^2 - xy + y^2 + 2x + 2y - 4$

$$\frac{\partial F}{\partial x} = 2x - y + 2, \quad \frac{\partial F}{\partial y} = -x + 2y + 2.$$

So we need to solve $2x - y + 2 = 0$ and $-x + 2y + 2 = 0$ to find the minima, maxima and saddle points.

① $2x - y + 2 = 0$ If we add $2\times$② to ① we get

② $-x + 2y + 2 = 0$ $0 + 3y + 6 = 0$, which we can solve to get $y = -2$.