$\frac{\partial F}{\partial x} = 2x + y$, $\frac{\partial F}{\partial y} = x + 2y$. At $P_0(-1,1)$, $\frac{\partial F}{\partial x} = -1$ & $\frac{\partial F}{\partial y} = 1$,

So gradient $= -\underset{\sim}{i} + \underset{\sim}{j}$

The direction in which the function increases most rapidly is

$-\underset{\sim}{i} + \underset{\sim}{j}$.

The direction in which the function decreases most rapidly is

$-(-\underset{\sim}{i} + \underset{\sim}{j}) = \underset{\sim}{i} - \underset{\sim}{j}$.

II) $F = \log_e xy + \log_e yz + \log_e zx$. At the point $P_0(1,1,1)$.

Gradient $= \frac{\partial F}{\partial x}\underset{\sim}{i} + \frac{\partial F}{\partial y}\underset{\sim}{j} + \frac{\partial F}{\partial z}\underset{\sim}{k}$ $\quad \frac{\partial F}{\partial x} = \frac{y}{xy} + \frac{z}{zx} = \frac{1}{x} + \frac{1}{x} = \frac{2}{x}$

$\frac{\partial F}{\partial y} = \frac{x}{xy} + \frac{z}{zy} = \frac{1}{y} + \frac{1}{y} = \frac{2}{y}$, $\quad \frac{\partial F}{\partial z} = \frac{y}{yz} + \frac{x}{xz} = \frac{1}{z} + \frac{1}{z} = \frac{2}{z}$

At $P_0(1,1,1)$, $\frac{\partial F}{\partial x} = 2$, $\frac{\partial F}{\partial y} = 2$, $\frac{\partial F}{\partial z} = 2$.

Gradient $= 2\underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}$

The direction in which the function increases most rapidly is

$2\underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}$.

The direction in which the function decreases most rapidly is

$-2\underset{\sim}{i} - 2\underset{\sim}{j} - 2\underset{\sim}{k}$

5) a) If $F(x,y,z) = x^2 + y^2 + z^2$, then the surface $x^2 + y^2 + z^2 = 3$ represents a surface of constant F. The normal to the surface is the direction in which F increases most rapidly.

So to calculate the normal, we need to calculate the gradient

$\frac{\partial F}{\partial x} = 2x$, $\frac{\partial F}{\partial y} = 2y$, $\frac{\partial F}{\partial z} = 2z$.

At $(1,1,1)$, $\frac{\partial F}{\partial x} = 2$, $\frac{\partial F}{\partial y} = 2$, $\frac{\partial F}{\partial z} = 2$

So the normal vector is $2\underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}$

To calculate the tangent plane, we work out what a vector based at $(1,1,1)$ would look like, take the dot product of it withthe normal vector and set the product equal to zero. We set it equal to zero because we want the normal vector and the vector based at $(1,1,1)$ to be perpendicular. ~~This will mean that~~ anyone