

$$= 200 \frac{\Delta r}{r} + 100 \frac{\Delta h}{h} = 200 \left(\frac{0.001}{0.05} \right) + 100 \left(\frac{0.001}{0.12} \right) = 200 \left(\frac{1}{50} \right) + 100 \left(\frac{1}{120} \right)$$
$$= 4 + \frac{10}{12} = 4 \frac{5}{6} \% = 4.8\bar{3} \%$$

3) I) $w = x^2 + y^2$, $x = \cos t$, $y = \sin t$

substituting For x & y
 $w = \cos^2 t + \sin^2 t$. However $\cos^2 t + \sin^2 t = 1$

so $w = 1$ and $\frac{dw}{dt} = 0$

II) $w = \frac{x}{z} + \frac{y}{z}$, $x = \cos^2 t$ & $y = \sin^2 t$. The question doesn't state

that z is a function, so we shall assume that it is an independent variable (ie that z is independent of t).

substituting For x & y

$$w = \frac{\cos^2 t}{z} + \frac{\sin^2 t}{z}$$

$\cos^2 t + \sin^2 t = 1$, so $w = \frac{1}{z}$

$$\frac{dw}{dt} = 0.$$

III) $w = 2ye^x - \log_e z$. $x = \log_e(t^2+1)$, $y = \tan^{-1}t$. As in (II), we shall assume that z is an independent variable (ie z is independent of t).

substituting For x & y

$$w = 2(\tan^{-1}t)(t^2+1) - \log_e z.$$

to work out $\frac{dw}{dt}$, we need to know that $\frac{d}{dt}(\tan^{-1}t) = \frac{1}{t^2+1}$

$$\frac{dw}{dt} = 2(t^2+1) \frac{d}{dt}(\tan^{-1}t) + 2(\tan^{-1}t) \frac{d}{dt}(t^2+1)$$

$$= 2(t^2+1) \frac{1}{t^2+1} + 2(\tan^{-1}t)(2t)$$

$$= 2 + 4t \tan^{-1}t$$

4) I) $F = x^2 + xy + y^2$. Find the gradient at $P_0(-1,1)$, along with the direction of steepest climb and steepest Fall.

The gradient is $\frac{\partial F}{\partial x} \mathbf{i} + \frac{\partial F}{\partial y} \mathbf{j} = \frac{\partial F}{\partial x} \mathbf{i}$