

Homework I

$$1) \quad T = x(e^y + e^{-y}) \quad x = 2, \Delta x = 0.1 \quad y = \log_e 2, \Delta y = 0.02$$

$$\Delta T = \frac{\partial T}{\partial x} \Delta x + \frac{\partial T}{\partial y} \Delta y + \frac{1}{2} \frac{\partial^2 T}{\partial x^2} \Delta x^2 + \frac{1}{2} \frac{\partial^2 T}{\partial y^2} \Delta y^2 + \frac{\partial^2 T}{\partial x \partial y} \Delta x \Delta y + \text{terms containing}$$

three or more Δx or Δy multiplied together.

Because Δx & Δy are small, we shall ignore all the terms contain two or more Δx or Δy multiplied together.

$$\Delta T = \frac{\partial T}{\partial x} \Delta x + \frac{\partial T}{\partial y} \Delta y$$

$$\frac{\partial T}{\partial x} = e^y + e^{-y}, \quad \frac{\partial T}{\partial y} = x(e^y - e^{-y})$$

$$\Delta T = (e^y + e^{-y}) \Big|_{y=\log_e 2} 0.1 + x(e^y - e^{-y}) \Big|_{\substack{x=2 \\ y=\log_e 2}} 0.02$$

$$\Delta T = (2 + \frac{1}{2}) 0.1 + 2(2 - \frac{1}{2}) 0.02$$

$$= (0.25) + (0.06) = 0.31$$

$$\Delta T = 0.31$$

$$2) \quad V = \pi r^2 h \quad r = 5.0 \text{ cm} = 0.050 \text{ m} \quad h = 12.0 \text{ cm} = 0.12 \text{ m}$$

Correct to the nearest millimetre implies that $\Delta r = 0.001 \text{ m}$, $\Delta h = 0.001 \text{ m}$.

$$\Delta V = \frac{\partial V}{\partial r} \Delta r + \frac{\partial V}{\partial h} \Delta h$$

The % error can be obtained From $\frac{\Delta V}{V} \times 100$

$$100 \frac{\Delta V}{V} = \frac{100}{V} \frac{\partial V}{\partial r} \Delta r + \frac{100}{V} \frac{\partial V}{\partial h} \Delta h$$

$$\frac{\partial V}{\partial r} = 2\pi r h, \quad \frac{\partial V}{\partial h} = \pi r^2 \quad \text{So} \quad \frac{100 \Delta V}{V} = \frac{100}{\pi r^2 h} 2\pi r h \Delta r + \frac{100}{\pi r^2 h} \pi r^2 \Delta h$$