261: 29th November

'C' Programming

Commands

When the compiler compiles a program, its task is divided into two parts. Firstly, the part of the compiler called the 'preprocessor' goes through the file and, amongst other things, removes the comments and tidies it up. The second part then converts the file into a program that you can run.

Commands beginning with the character '#' have a special place in the 'C' programming language. They are instructions for the preprocessor. For example, when we put `#include<stdio.h>` at the beginning of the program, it is an instruction for the preprocessor to fetch the file called `stdio.h` and insert it at the beginning of the file.

Another command that is commonly used is `#define`. If we put `#define N 4` at the beginning of the file, then, when the preprocessor goes through the file, it will replace every occurrance of `N` with the number `4`. The second part of the compiler will then run through and only see the number `4`, where before we had put the variable `N`.

Effectively, this is a way of declaring a variable, when the value of the variable won't change during the execution of the program.

When we have syntax errors in our program, such as a missed semicolon or bracket, and the compiler gives us an error message, the message is generated by the second part of the program. The second part is responsible for making sure that the program is correctly written. However, because the preprocessor substitutes `N` for `4`, before the second part sees the program, the second part doesn't see the `4` or the `N` as a variable, it just sees the number. For this reason, `N` doesn't have to be declared in the way that a normal variable would be.

Incidently, it's considered good programming practise, in `#define` commands, to name the variables in captials.

How to Solve a Matrix Equation

To solve any matrix equation, we must first transform the equation to make the matrix upper diagonal. We can then employ backwards substitution to solve the equation. The diaginalization step is known as gaussian elimination.

As an example, let us start with the equation

$$\begin{pmatrix} 2 & 3 & 1 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 10 \\ 26 \\ 41 \end{pmatrix}$$

and divide the top row by the top left entry. We do this to make the first term on the top row equal to 1. This gives,

$$\begin{pmatrix} 1 & \frac{3}{2} & \frac{1}{2} \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 26 \\ 41 \end{pmatrix}$$

Now, by subtracting 4*(top row) from the second row, the left entry on the second row will become zero. Similary, by subtracting 7*(top row) from the bottom row, the left entry of the bottom row will become zero. Doing the same to the right hand side, this gives us

$$\begin{pmatrix} 1 & \frac{3}{2} & \frac{1}{2} \\ 0 & -1 & 4 \\ 0 & -\frac{5}{2} & \frac{11}{2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \\ 6 \end{pmatrix}$$

The next step is to divide the second row by the middle element. This will make the middle element equal to 1.

$$\begin{pmatrix} 1 & \frac{3}{2} & \frac{1}{2} \\ 0 & 1 & -4 \\ 0 & -\frac{5}{2} & \frac{11}{2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ -6 \\ 6 \end{pmatrix}$$

We subtract $-\frac{5}{2}$*(row two) from the third row.

$$\begin{pmatrix} 1 & \frac{3}{2} & \frac{1}{2} \\ 0 & 1 & -4 \\ 0 & 0 & -\frac{9}{2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ -6 \\ -9 \end{pmatrix}$$

Which is our equation in upper diagonal form. Now, we use backward substitution to calculate the values of x_1, x_2 and x_3.

From the bottom row, $-\frac{9}{2}x_3 = -9$. So $x_3 = 2$. Now we use x_3 in the second line, to calculuate x_2. The second line tells us that

$$x_2 - 4x_3 = -6$$

Substituting for x_3, we have,

$$x_2 - 8 = -6$$

which tells us that $x_2 = 2$.

Finally, we use x_2 and x_3 to work out the value of x_1. From the first line, we have that,

$$x_1 + \frac{3}{2}x_2 + \frac{1}{2}x_3 = 5$$

Substituting for x_2 and x_3, we have that,

$$x_1 + 3 + 1 = 5$$

This can be rearranged to give $x_1 = 1$. So, the solution is that

$$x = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$