Assignment 6

MA341C — Seminar on *Proofs from THE BOOK*
Trinity College Dublin

NAME AND SURNAME: ....

STUDENT NUMBER: .... NUMBER OF PAGES: ....

Note: solutions to this assignment are due by 11am on Wednesday, October 31st. Please attach a cover sheet with a declaration (http://tcd-ie.libguides.com/plagiarism/declaration) confirming that you know and understand College rules on plagiarism. All exercises are weighed equally unless otherwise stated.

Exercise 1. Prove that it is impossible to draw an uncountable family of pairwise disjoint figure-eight-shapes in $\mathbb{R}^2$ (a figure-eight-shape are two circles tangent to each other which are in different half-planes relative to the tangent line).

Exercise 2. Two points A and B are chosen on the unit circle.

1. For the given n, what is the maximal perimeter of an n-gon inscribed into the unit circle for which the segment AB is one of its sides?
2. Same question if we replace the words "maximal perimeter" by "maximal area".

Exercise 3. Show that for every prime $p > 2$, there exists a non-Abelian finite group G for which all non-unit elements are of order p.

Exercise 4.

1. Show that the function

$$\phi(x) = \begin{cases} e^{-1/x^2}, x > 0, \\ 0, \quad x \leq 0 \end{cases}$$

is smooth (i.e. has derivatives of all orders at all $x \in \mathbb{R}$), and that $\phi^{(n)}(0) = 0$ for all $n \geq 0$.

2. Let $\{a_n\}$ be a sequence of real numbers. Show that there exists a smooth function $\psi(x)$ such that $\psi^{(n)}(0) = a_n$ for all $n \geq 0$.