Assignment 10

MA341C — Seminar on *Proofs from THE BOOK*

Trinity College Dublin

NAME AND SURNAME: ....

STUDENT NUMBER: .... NUMBER OF PAGES: ....

Note: solutions to this assignment are due by 11am on Wednesday, November 28th. Please attach a cover sheet with a declaration (http://tcd-ie.libguides.com/plagiarism/declaration) confirming that you know and understand College rules on plagiarism. All exercises are weighed equally unless otherwise stated.

Exercise 1. Prove the identity $\sum_{i=0}^{k}\binom{n}{i}\binom{m}{k-i}=\binom{n+m}{k}$. Deduce that $\binom{2n}{n}$ is a sum of $n+1$ squares.

Exercise 2. A convex polygon P has all its vertices on the integer grid in $\mathbb{R}^2$, and no side of P is a grid line. Prove that the sum of lengths of horizontal grid lines inside P is equal to the sum of lengths of vertical grid lines inside P.

Exercise 3. Let G be a simple planar graph with 11 vertices. Show that its complement G^c is non-planar.

Exercise 4. In this question, we consider graphs embedded in $\mathbb{R}^2$ in such a way that edges may intersect, but the number of intersection points is finite, and each edge only contains the two vertices it is adjacent to.

1. For two graphs G and H in $\mathbb{R}^2$ such that no edge of either of them contains any vertices of the other, we define $\mathrm{in}(G,H)$ to be the modulo 2 class of the number of intersection points between edges of G and edges of H. Prove that $\mathrm{in}(G,H)=0$ if G and H are cycles. (*Hint*: use the Jordan curve theorem.)
2. The self-intersection number $\mathrm{in}(G)$ is the modulo 2 class of the number of intersections between edges of G at their interior points (so that intersections of adjacent edges do not count). Suppose that for any edge e of G, the edges not adjacent to e form a cycle. Show that in this case $\mathrm{in}(G)$ does not depend on the way G is embedded in the plane.
3. Show that $K_{3,3}$ and K_5 satisfy the condition of the previous question, and then conclude they are not planar by computing their self intersection number.