

2BA1: Mathematics for Students in Computer Science
Homework problems due May 2, 2008

1. For the 3-periodic sequence of complex numbers

$$\mathbf{a} = \{\dots, 2 + i, 0, 1 - i, 2 + i, \dots\}$$

(that is, $\mathbf{a}_0 = 2 + i$, $\mathbf{a}_1 = 0$ etc.), compute its discrete Fourier transform, its convolution with itself $\mathbf{a} \star \mathbf{a}$, and the discrete Fourier transform of $\mathbf{a} \star \mathbf{a}$.

2. Compute the product of quaternions $2 - i + 7j$ and $1 + i + k$.
3. For vectors $\mathbf{u} = (6, 2, -1)$, $\mathbf{v} = (2, -1, 1)$, and $\mathbf{w} = (-1, -1, 1)$, compute (\mathbf{u}, \mathbf{w}) , $\mathbf{v} \times \mathbf{u}$, and $(\mathbf{u}, \mathbf{v} \times (\mathbf{u} \times \mathbf{w}))$.
4. Find the image of the point $(1, 1, 1)$ in 3-space under the rotation through $\frac{2\pi}{3}$ about the line connecting the origin with $(9, 2, 6)$.
5. (a) List all numbers between 0 and 24 that are coprime to 24.
(b) For any number k between 0 and 24 that is coprime to 24, find a number l such that $kl \equiv 1 \pmod{24}$.
6. Show that for any integers a and b ,

$$\gcd(a, b) = \gcd(5a + 12b, 12a + 29b).$$

7. (a) Use the Euclidean algorithm to compute the greatest common divisor of 357 and 239.
(b) Describe all integer solutions to $357x - 239y = 4$.
(c) Find all integers n congruent to 1 modulo 357 and to 67 modulo 239.
In other words, solve the system of congruences

$$\begin{cases} n \equiv 1 \pmod{357}, \\ n \equiv 67 \pmod{239}. \end{cases}$$