

MA2316: quiz after the study week

A complete solution to each of the questions below earns you 1% towards the continuous assessment mark. If all solutions are correct, you will get a 1% bonus, so 4% in total. Please do not forget to put your name and student number on your script.

1. Which of the following systems of congruences have solutions? Explain your answer, and find all solutions.

$$(a) \begin{cases} x \equiv 11 \pmod{84}, \\ x \equiv 8 \pmod{36}. \end{cases} \quad (b) \begin{cases} x \equiv 11 \pmod{84}, \\ x \equiv 7 \pmod{36}. \end{cases} \quad (c) \begin{cases} x \equiv 11 \pmod{84}, \\ x \equiv 23 \pmod{36}. \end{cases}$$

2. Which of the following congruences have solutions? Explain your answer.

(a) $x^2 - 10x + 11 \equiv 0 \pmod{61}$;

(b) $x^2 - 10x + 11 \equiv 0 \pmod{183}$;

(c) $x^2 + 2x - 9 \equiv 0 \pmod{97}$.

3. Consider the polynomial $f(x) = x^3 + 3x + 9$. It can be directly checked that its roots in $\mathbb{Z}/5\mathbb{Z}$ are $x = 3$ and $x = 4$. How many roots does this polynomial have in $\mathbb{Z}/25\mathbb{Z}$? Write down those roots.

MA2316: quiz after the study week

A complete solution to each of the questions below earns you 1% towards the continuous assessment mark. If all solutions are correct, you will get a 1% bonus, so 4% in total. Please do not forget to put your name and student number on your script.

1. Which of the following systems of congruences have solutions? Explain your answer, and find all solutions.

$$(a) \begin{cases} x \equiv 11 \pmod{84}, \\ x \equiv 8 \pmod{36}. \end{cases} \quad (b) \begin{cases} x \equiv 11 \pmod{84}, \\ x \equiv 7 \pmod{36}. \end{cases} \quad (c) \begin{cases} x \equiv 11 \pmod{84}, \\ x \equiv 23 \pmod{36}. \end{cases}$$

2. Which of the following congruences have solutions? Explain your answer.

(a) $x^2 - 10x + 11 \equiv 0 \pmod{61}$;

(b) $x^2 - 10x + 11 \equiv 0 \pmod{183}$;

(c) $x^2 + 2x - 9 \equiv 0 \pmod{97}$.

3. Consider the polynomial $f(x) = x^3 + 3x + 9$. It can be directly checked that its roots in $\mathbb{Z}/5\mathbb{Z}$ are $x = 3$ and $x = 4$. How many roots does this polynomial have in $\mathbb{Z}/25\mathbb{Z}$? Write down those roots.