MA 1112: Linear Algebra II
Tutorial problems, March 12, 2019

1. For the space $\mathbb{R}^3$ with the standard scalar product, find the orthogonal basis e_1, e_2, e_3 obtained by Gram–Schmidt orthogonalisation from $f_1 = \begin{pmatrix}1\\3\\3\end{pmatrix}$, $f_2 = \begin{pmatrix}1\\2\\4\end{pmatrix}$, $f_3 = \begin{pmatrix}1\\1\\1\end{pmatrix}$.

2. Show that the formula

$$(\begin{pmatrix}x_1\\y_1\end{pmatrix}, \begin{pmatrix}x_2\\y_2\end{pmatrix}) = x_1x_2 + \frac{1}{2}(x_1y_2 + x_2y_1) + y_1y_2.$$

defines a scalar product on $\mathbb{R}^2$, and find an orthonormal basis of $\mathbb{R}^2$ with respect to that scalar product.

3. For the vector space of all polynomials in t of degree at most 3 and the scalar product on this space given by

$$(p(t), q(t)) = \int_{-1}^{1} p(t)q(t)\,dt,$$

find the result of Gram–Schmidt orthogonalisation of the vectors $1, t, t^2, t^3$.

Optional question: Show that in $\mathbb{R}^n$, it is impossible to find $n+2$ vectors that only form obtuse angles (that is, $(v_i, v_j) < 0$ for all $i \neq j$).