

MA 1112: Linear Algebra II
Tutorial problems, January 29, 2019

Consider two subspaces of \mathbb{R}^4 : the subspace \mathbf{U}_1 which is the linear span of the vectors

$$\begin{pmatrix} 0 \\ 3 \\ -2 \\ 2 \end{pmatrix}, \begin{pmatrix} -9 \\ 8 \\ 2 \\ -3 \end{pmatrix}, \begin{pmatrix} 4 \\ 1 \\ 1 \\ 1 \end{pmatrix},$$

and the subspace \mathbf{U}_2 which is the linear span of the vectors

$$\begin{pmatrix} 6 \\ 0 \\ -3 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 3 \\ 0 \\ 5 \end{pmatrix}, \begin{pmatrix} 9 \\ -1 \\ -5 \\ 0 \end{pmatrix}.$$

1. Find a basis of \mathbf{U}_1 and a basis of \mathbf{U}_2 .
2. Find a basis for the intersection $\mathbf{U}_1 \cap \mathbf{U}_2$.
3. Find a basis of \mathbf{U}_1 relative to $\mathbf{U}_1 \cap \mathbf{U}_2$.

4. Is the subspace spanned by the vectors $\mathbf{v}_1 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ and $\mathbf{v}_2 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

invariant under the linear transformation φ of \mathbb{R}^3 that multiplies every vector by the matrix $\mathbf{A} = \begin{pmatrix} -4 & 4 & 5 \\ 16 & 2 & -6 \\ -16 & 1 & 9 \end{pmatrix}$? Explain your answer.