MA 1111: Linear Algebra I
Tutorial problems, November 22, 2018

1. The set of all complex numbers forms a 2-dimensional vector space over real numbers with a basis $1, i$. Compute, relative to this basis, the matrix of the linear operator on that space which maps every complex number z to $(3-7i)z$.

2. Let $V = \mathbb{R}^2$, $e_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $e_2 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ a basis of V, $\varphi \colon V \to V$ a linear transformation whose matrix $A_{\varphi,\mathbf{e}}$ relative to the basis e_1, e_2 is $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

(**a**) Find the transition matrix $M_{\mathbf{ef}}$ from the basis e_1, e_2 to the basis $f_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$, $f_2 = \begin{pmatrix} 4 \\ -9 \end{pmatrix}$, and compute the matrix $A_{\varphi,\mathbf{f}}$.

(**b**) Compute the matrix $A_{\varphi,\mathbf{v}}$ of the linear transformation φ relative to the basis of standard unit vectors $v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $v_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

3. Does there exist a change of basis making the matrix $A = \begin{pmatrix} 3 & 0 & -1 \\ 5 & -1 & -8 \\ -1 & 1 & 4 \end{pmatrix}$ diagonal? Why?

4. For the matrix $A = \begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$, find a closed formula for A^n.