MA 1111: Linear Algebra I
Tutorial problems, November 15, 2018

1. (**a**) For the vector space $\mathbb{R}^3$, show that the vectors

$$e_1 = \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}, e_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, e_3 = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

form a basis, and so do

$$f_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, f_2 = \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}, f_3 = \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}.$$

(**b**) For the vector space $\mathbb{R}^3$, find the transition matrix $M_{\mathbf{e},\mathbf{f}}$ for the two bases from the previous question.

(**c**) Given that a vector has coordinates $1, 4, -3$ with respect to the basis f_1, f_2, f_3, find its coordinates with respect to the basis e_1, e_2, e_3.

(**d**) Given that a vector has coordinates $1, 4, -3$ with respect to the basis e_1, e_2, e_3, find its coordinates with respect to the basis f_1, f_2, f_3.

In the following two questions, we denote by P_n the vector space of all polynomials in x of degree at most n.

2. Write down the transition matrix between the bases 1, x, ..., x^n and 1, $x+1$, ..., $(x+1)^n$ of P_n for (**a**) $n = 1$; (**b**) $n = 2$; (**c**) $n = 3$.

3. Which of the following functions from P_3 to P_3 are linear operators? Explain your answers. When a function is a linear operator, write down its matrix relative to the standard basis $1, x, x^2, x^3$.

(**a**) $f(x) \mapsto \frac{f(x)-f(0)}{x}$; (**b**) $f(x) \mapsto xf'(x) - 2f(x)$; (**c**) $f(x) \mapsto f''(x)f'(x) - t^2 f'''(x)$.