MA 1111: Linear Algebra I
Tutorial problems, October 19, 2018

In problems 1–4, determine whether the system of vectors $\{v_i\}$ in the vector space V (a) is linearly independent; (b) complete; (c) forms a basis.

1. $V = \mathbb{R}^2$, $v_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$, $v_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

2. $V = \mathbb{R}^2$, $v_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$, $v_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $v_3 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

3. $V = \mathbb{R}^3$, $v_1 = \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$, $v_2 = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$, $v_3 = \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$, $v_4 = \begin{pmatrix} 3 \\ -5 \\ 2 \end{pmatrix}$, $v_5 = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$.

4. $V = \mathbb{R}^3$, $v_1 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$, $v_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $v_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$.

5. (**a**) Prove that if the system of vectors u, v, w of some vector space is linearly independent, then the system of vectors $u-2w$, $v+w$, w is linearly independent as well.

(**b**) Prove that if the system of vectors u, v, w of some vector space is complete, then the system of vectors $u-2w$, $v+w$, w is complete as well.