MA 1111: Linear Algebra I
Homework problems for November 16, 2018

Solutions to this problem sheet are to be handed in after our class at 1pm on Friday. Please attach a cover sheet with a declaration `http://tcd-ie.libguides.com/plagiarism/declaration` confirming that you know and understand College rules on plagiarism. On the same cover sheet, please put your name, student number, and name of the degree (Maths/TP/TSM), and staple all the sheets together. (Failure to do that may result in misplaced/lost sheets, for which no responsibility can be taken by instructors.)

1. True or false (prove true statements and give counterexamples to false ones):

(**a**) if a system of vectors in some vector space contains a vector which is equal to a linear combination of other vectors from the same system, then these vectors are linearly dependent;

(**b**) if a system of vectors in some vector space is linearly dependent, then it contains a vector which is equal to a linear combination of other vectors from this system;

(**c**) if a system of vectors in some vector space is linearly dependent, then every vector from this system is equal to a linear combination of other vectors from this system.

2. Show that the vectors $e_1 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}, e_2 = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$ form a basis of $\mathbb{R}^2$, and compute coordinates of $\mathbf{v} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ relative to that basis.

3. For $V = \mathbb{R}^3$, find the transition matrix M_{ef} from the basis $e_1 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, e_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, e_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ to the basis $f_1 = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}, f_2 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, f_3 = \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}$, and find the coordinates of the vector $\begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix}$ in $\mathbb{R}^3$ relative to each of the bases e_1, e_2, e_3 and f_1, f_2, f_3.

4. Let $v = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \in \mathbb{R}^3$.

(**a**) Show that the function from $\mathbb{R}^3$ to $\mathbb{R}^3$ given by

$$w \mapsto v \times w$$

is a linear operator, and find its matrix relative to the basis of standard unit vectors.

(**b**) Show that the function from $\mathbb{R}^3$ to $\mathbb{R}^1$ given by

$$w \mapsto v \cdot w$$

is a linear transformation, and find its matrix relative to the bases of standard unit vectors in $\mathbb{R}^3$ and $\mathbb{R}^1$.