MA 1111: Linear Algebra I
Homework problems for November 9, 2018

Solutions to this problem sheet are to be handed in after our class at 1pm on Friday. Please attach a cover sheet with a declaration `http://tcd-ie.libguides.com/plagiarism/declaration` confirming that you know and understand College rules on plagiarism. On the same cover sheet, please put your name, student number, and name of the degree (Maths/TP/TSM), and staple all the sheets together. (Failure to do that may result in misplaced/lost sheets, for which no responsibility can be taken by instructors.)

1. For the system

$$\begin{cases} 2x_1 - 4x_2 + x_3 + x_4 = 0, \\ x_1 - 2x_2 + 5x_4 = 0, \end{cases}$$

find some vectors $v_1, \ldots, v_k$ such that the solution set to this system equals $\operatorname{span}(v_1, \ldots, v_k)$.

2. For the matrix $A = \begin{pmatrix} 1 & -2 & -1 & 1 \\ 2 & -1 & -2 & 1 \\ 0 & 6 & 0 & -2 \end{pmatrix}$, find some vectors $v_1, \ldots, v_k$ such that the solution set to the system $Ax = 0$ equals $\operatorname{span}(v_1, \ldots, v_k)$.

3. (**a**) Prove that if V is a vector space, $v \in V$, $c \in \mathbb{R}$, and $c \cdot v = 0$, then $c = 0$ or $v = 0$.

(**b**) Show that for every vector space V and every element $\mathbf{v} \in V$ the opposite element is unique: if $\mathbf{v} + \mathbf{x} = 0$, then $\mathbf{x} = -\mathbf{v}$.

(**c**) Show that for every vector space V and every element $\mathbf{v} \in V$ we have $(-1) \cdot \mathbf{v} = -\mathbf{v}$.

4. Which of the following subsets U of $\mathbb{R}^3$ are subspaces? Explain your answers.

(**a**) $U = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : 3x - 5y + z = 0 \right\}$.

(**b**) $U = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : 2x - y + z = 1 \right\}$.

(**c**) $U = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : x^2 - y^2 + z^2 = 0 \right\}$.

(**d**) U is the solution set to the system of linear equations $A\mathbf{x} = 0$, where $A = \begin{pmatrix} 3 & 4 & 61 \\ 112 & -1 & 34 \\ 109 & -5 & -27 \end{pmatrix}$.

5. Which of the following subsets of the vector space of all polynomials in one variable x are subspaces? Explain your answers.

(**a**) $U = \{f(x) \colon f(1) = 0\}$;

(**b**) $U = \{f(x) \colon f(1) = f(2) = 0\}$;

(**c**) $U = \{f(x) \colon f(1) + f(2) = 0\}$;

(**d**) $U = \{f(x) \colon f(1) \cdot f(2) = 0\}$.