MA 1111: Linear Algebra I
Homework problems for November 2, 2018

Solutions to this problem sheet are to be handed in after our class at 1pm on Friday. Please attach a cover sheet with a declaration

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confirming that you know and understand College rules on plagiarism. On the same cover sheet, please put your name, student number, and name of the degree (Maths/TP/TSM), and staple all the sheets together. (Failure to do that may result in misplaced/lost sheets, for which no responsibility can be taken by instructors.)

1. For the following permutations determine whether they are odd or even (the answer depends on n):
(**a**) $\begin{pmatrix} 1 & 2 & 3 & \dots & n-1 & n \\ n & 1 & 2 & \dots & n-2 & n-1 \end{pmatrix}$; (**b**) $\begin{pmatrix} 1 & 2 & 3 & \dots & n-2 & n-1 & n \\ n & n-1 & n-2 & \dots & 3 & 2 & 1 \end{pmatrix}$.

2. (**a**) Without directly evaluating the determinant, show that for each choice of α, β, and γ the matrix

$$\begin{pmatrix} \cos^2\alpha & \cos^2\beta & \cos^2\gamma \\ \sin^2\alpha & \sin^2\beta & \sin^2\gamma \\ 1 & 1 & 1 \end{pmatrix}$$

is not invertible.

(**b**) Let A be a square matrix. Prove that A is invertible if and only if AA^T is invertible.

3. Using the property $\mathrm{tr}(AB) = \mathrm{tr}(BA)$ proved in an earlier homework, prove that

(**a**) there do not exist $n \times n$-matrices P and Q such that $PQ - QP = I_n$,

(**b**) if for two $n \times n$-matrices P and Q we have $PQ - QP = P$, then the matrix P is not invertible.

4. For each of the following systems of vectors in $\mathbb{R}^3$, find out whether it is linearly independent:

(**a**) $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$; (**b**) $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$;

(**c**) $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$; (**d**) $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

5. For each of the following systems of vectors in $\mathbb{R}^3$, find out whether it is complete:

(**a**) $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$; (**b**) $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$;

(**c**) $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$; (**d**) $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.