MA 1111: Linear Algebra I
Homework problems for October 5, 2018

Solutions to this problem sheet are to be handed in after our class at 1pm on Friday. Please attach a cover sheet with a declaration

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confirming that you know and understand College rules on plagiarism. On the same cover sheet, please put your name, student number, and name of the degree (Maths/TP/TSM), and staple all the sheets together. (Failure to do that may result in misplaced/lost sheets, for which no responsibility can be taken by instructors.)

1. For each of the following choices of matrices A and B find out which of the matrices $A+B$, BA, and AB are defined, and compute those which are defined:

(**a**) $A = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}$, $B = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$;

(**b**) $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$;

(**c**) $A = \begin{pmatrix} 5 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 4 \\ 0 & 1 \\ 2 & 5 \end{pmatrix}$;

(**d**) $A = \begin{pmatrix} 2 & 4 \\ 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 4 \\ 2 & 2 \end{pmatrix}$.

2. Which of the following matrices are invertible? Compute inverses for them. (**a**) $\begin{pmatrix} 5 & 3 \\ 2 & 1 \end{pmatrix}$; (**b**) $\begin{pmatrix} 6 & 3 \\ 2 & 1 \end{pmatrix}$; (**c**) $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 3 & 9 \end{pmatrix}$; (**d**) $\begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \end{pmatrix}$.

3. (**a**) Show that if A and B are matrices for which both products AB and BA are defined, then both products AB and BA are square matrices (maybe of different sizes).

(**b**) For an $n \times n$-matrix A, its *trace* $\mathrm{tr}(A)$ is defined as the sum of diagonal elements,

$$\mathrm{tr}(A) = A_{11} + A_{22} + \cdots + A_{nn}.$$

Show that if U is an $n \times m$-matrix, and V is an $m \times n$-matrix, then $\mathrm{tr}(UV) = \mathrm{tr}(VU)$. Explain why this does not contradict the example from class where we found two 2×2-matrices for which $UV \neq VU$.

4. (**a**) Give an example of a 2×3-matrix A and a 3×2-matrix B for which $AB = I_2$. (*Hint*: in this case, there is already an example with matrices with entries entries from $\{0, 1\}$).

(**b**) Show that there do not exist an example of a 3×2-matrix A and a 2×3-matrix B for which $AB = I_3$.