MA 1111: Linear Algebra I

Homework problems for September 21, 2018

Solutions to this problem sheet are to be handed in after our class at 1pm on Friday. Please attach a cover sheet with a declaration

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confirming that you know and understand College rules on plagiarism. On the same cover sheet, please put your name, student number, and name of the degree (Maths/TP/TSM), and staple all the sheets together. (Failure to do that may result in misplaced/lost sheets, for which no responsibility can be taken by instructors.)

1. (**a**) Does the straight line passing through the points $(6, 8)$ and $(9, 13)$ contain the point $(1, 0)$? (**b**) Do the points $(1, 1)$, $(4, 5)$, and $(9, -5)$ form a right triangle?

In the next two questions, we consider the vectors

$$\mathbf{u} = (1, -1, 1), \quad \mathbf{v} = (2, 3, -1), \quad \mathbf{w} = (0, 2, 1).$$

2. Compute the following products:

$$\mathbf{u} \cdot \mathbf{v}, \quad \mathbf{v} \cdot \mathbf{w}, \quad \mathbf{v} \times \mathbf{w}, \quad \mathbf{u} \times \mathbf{w}, \quad \mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}), \quad \mathbf{v} \cdot (\mathbf{u} \times \mathbf{w}).$$

3. Use your results from the previous question to compute (**a**) the area of the parallelogram determined by the vectors **u** and **w**; (**b**) the volume of the parallelepiped determined by **u**, **v**, and **w**.

4. Prove that the coordinates of the point (x', y') where the [counter-clockwise] rotation through α about the point $(0, 0)$ brings the given point (x, y) are

$$x' = x \cos \alpha - y \sin \alpha,$$
$$y' = x \sin \alpha + y \cos \alpha.$$

(*Hint*: show that for the points $(x, y) = (1, 0)$ and $(x, y) = (0, 1)$ directly, and then use the fact that the vector from the origin to the point (x, y) is equal to the vector $x \cdot (1, 0) + y \cdot (0, 1)$.)