MA 1111: Linear Algebra I

Selected answers/solutions to the assignment for November 9, 2018

1. This system is of the form $Ax = 0$, where $A = \begin{pmatrix} 2 & -4 & 1 & 1 \\ 1 & -2 & 0 & 5 \end{pmatrix}$. The reduced row echelon form of the matrix A is $\begin{pmatrix} 1 & -2 & 0 & 5 \\ 0 & 0 & 1 & -9 \end{pmatrix}$, , so x_2 and x_4 are free variables, and the general solution corresponding to the parameters $x_2 = s$, $x_4 = t$ is $\begin{pmatrix} 2s-5t \\ s \\ 9t \\ t \end{pmatrix} = s \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} -5 \\ 0 \\ 9 \\ 1 \end{pmatrix}$.

Therefore we can take $v_1 = \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ and $v_2 = \begin{pmatrix} -5 \\ 0 \\ 9 \\ 1 \end{pmatrix}$.

2. The reduced row echelon form of this matrix is $\begin{pmatrix} 1 & 0 & -1 & 1/3 \\ 0 & 1 & 0 & -1/3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$, so x_3 and x_4 are free variables, and the general solution corresponding to the parameters $x_3 = s$, $x_4 = t$ is $\begin{pmatrix} s - \frac{1}{3}t \\ \frac{1}{3}t \\ s \\ t \end{pmatrix} = s \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} -\frac{1}{3} \\ \frac{1}{3} \\ 0 \\ 1 \end{pmatrix}$. Therefore we can take $v_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ and $v_2 = \begin{pmatrix} -\frac{1}{3} \\ \frac{1}{3} \\ 0 \\ 1 \end{pmatrix}$.

3. (**a**) If $c = 0$, there is nothing to prove. If $c \neq 0$, we can multiply our equality by c^{-1}, obtaining $0 = c^{-1} \cdot 0 = c^{-1}(cv) = (c^{-1}c)v = 1 \cdot v = v$.

(**b**) If $v + x = 0$, we have $-v = (-v) + 0 = (-v) + (v + x) = ((-v) + v) + x = 0 + x = x$, so $x = -v$.

(**c**) We have $v + (-1) \cdot v = 1 \cdot v + (-1) \cdot v = (1 + (-1)) \cdot v = 0 \cdot v = 0$, so $x = (-1) \cdot v$ satisfies $v + x = 0$, and by the previous question, $(-1) \cdot v = -v$.

4. (**a**) Yes, it is obviously closed under addition and taking scalar multiples.

(**b**) No (for example, 0 does not belong to U).

(**c**) No (for example, $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ belong to U, but their sum $\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$ does not belong to U).

(**d**) Yes, every solution set to a homogeneous system of linear equations is a subspace.

5. (**a**) Yes, if $f(1) = 0$ and $g(1) = 0$, then $(f+g)(1) = f(1)+g(1) = 0$, and $(cf)(1) = cf(1) = 0$.

(**b**) Yes, similar to the previous one.

(**c**) Yes, similar to the previous one.

(**d**) No (for example, $t-1$ and $t-2$ belong to U, but their sum $2t-3$ does not belong to U).