MA 1111: Linear Algebra I
Tutorial problems, October 7, 2015

1. $\mathbf{u} \times (\mathbf{v} \times \mathbf{w})$ is certainly defined, computing this vector amounts to computing vector products twice.

$\mathbf{v} \times (\mathbf{u} \cdot \mathbf{w})$ is not defined: vector products are defined for two vectors.

$\mathbf{u} \times \mathbf{v} \times \mathbf{w}$ is not defined: we only know how to compute vector products of two vectors, and depending on the choice of bracketings, $(\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$ and $\mathbf{u} \times (\mathbf{v} \times \mathbf{w})$ leads to two generally different results.

$(\mathbf{u} \cdot \mathbf{w}) \cdot \mathbf{v}$ is, superficially, not defined because the dot product is defined for two vectors. However, there is a convention to write $c \cdot \mathbf{v}$ for $c\mathbf{v}$, if c is a scalar and $\mathbf{v}$ is a vector, and if this convention is adopted, it is OK (see also notation in the hint to question 2 which uses that convention).

$\mathbf{u} \cdot (\mathbf{w} \cdot \mathbf{v})$ is, contrary to the previous one, never defined, since we can form the expression $c\mathbf{v}$ but not $\mathbf{v}c$; scalars always go before vectors.

$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$ is defined: it is a scalar product of two vectors one of which is a vector product of two vectors.

$\mathbf{u} \cdot \mathbf{v} \cdot \mathbf{w}$ is not defined, since depending on the choice of bracketings we get some very different things, one defined and the other not.

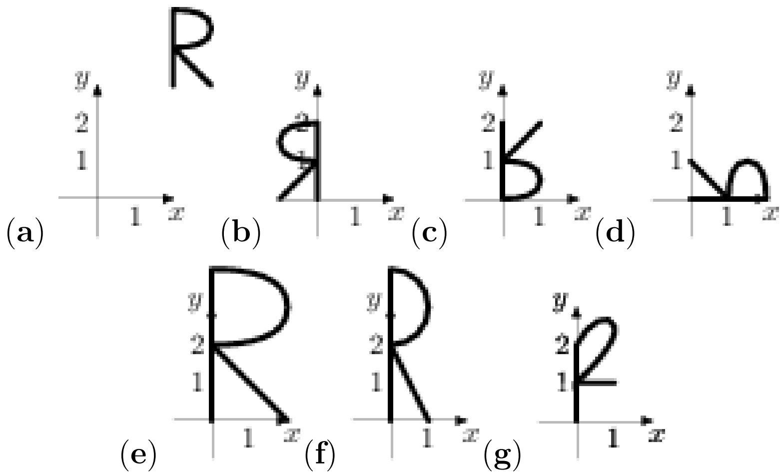
2. We have

$$\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) + \mathbf{v} \times (\mathbf{w} \times \mathbf{u}) + \mathbf{w} \times (\mathbf{u} \times \mathbf{v}) = (\mathbf{u} \cdot \mathbf{w}) \cdot \mathbf{v} - (\mathbf{u} \cdot \mathbf{v}) \cdot \mathbf{w} + (\mathbf{v} \cdot \mathbf{u}) \cdot \mathbf{w} - (\mathbf{v} \cdot \mathbf{w}) \cdot \mathbf{u} + (\mathbf{w} \cdot \mathbf{v}) \cdot \mathbf{u} - (\mathbf{w} \cdot \mathbf{u}) \cdot \mathbf{v} = 0$$

(because $\mathbf{u} \cdot \mathbf{w} = \mathbf{w} \cdot \mathbf{u}$ etc.).

3. Substituting $x = 1 - ay$ in the second equation, we get $a(1-ay)+y = 0$, so $a - a^2y + y = 0$, or in other words $a = y(a^2 - 1)$. Therefore for $a = \pm 1$ there are no solutions, and for $a \neq \pm 1$ there is just one solution $y = \frac{a}{a^2-1}$, $x = 1 - ay = \frac{-1}{a^2-1}$.

4.



(**h**) the transformation is $(x, y) \mapsto (1 - x, y)$.