Practice exam questions for the module 1111, Michaelmas 2015

Solutions to this problem sheet will be discussed as a part of the revision class on Thursday December 17.

1. Is the vector $\mathbf{n} = (-1, -3, 4)$ parallel to the intersection line of the plane α passing through the points $(0, 1, -1)$, $(5, 1, -3)$, and $(2, -3, 3)$ and the plane β passing through the points $(2, 1, -1)$, $(5, 1, -3)$, and $(1, -2, 3)$? Why?

2. Consider the system of linear equations

$$\begin{cases} 4x_1 - 2x_2 + 2x_3 = 5, \\ 2x_1 - x_2 + 3x_3 = 2, \\ 7x_1 - 2x_2 + 3x_3 = 6. \end{cases}$$

(**a**) Use Gaussian elimination to solve this system.

(**b**) Compute the inverse matrix using the adjugate matrix formula, and use it to solve this system.

3. Find i, j, and k for which the product $a_{61}a_{i6}a_{1j}a_{25}a_{54}a_{3k}$ occurs in the expansion of the 6×6-determinant with coefficient (-1).

4. Determine all values of x for which the three vectors

$$\begin{pmatrix} 2-x \\ -1 \\ 5 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ -x \\ 5 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 1 \\ 3-x \end{pmatrix}$$

are linearly dependent.

5. Let us consider matrices B which commute with the matrix $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, that is $AB = BA$. How many rows and columns may such matrix have? Show that the set of all such matrices forms a vector space with respect to the usual matrix operations. Compute the dimension of that space.

6. Show that the map of the space P_2 of all polynomials in x of degree at most 2 to the same space that takes every polynomial $f(x)$ to $3x^2f''(x) + 3f(x-1)$ is a linear transformation, and compute the matrix of that transformation relative to the basis $1, x+1, (x+1)^2$.