MA 1111: Linear Algebra I
Homework problems due December 3, 2015

Solutions to this problem sheet are to be handed in after our class at 3pm on Thursday. Please attach a cover sheet with a declaration `http://tcd-ie.libguides.com/plagiarism/declaration` confirming that you know and understand College rules on plagiarism.

Please put your name and student number on each of the sheets you are handing in. (Or, ideally, staple them together).

1. For each of the following subsets of the set of all polynomials in one variable find out whether this subset is a vector space (with usual operations on polynomials).

(**a**) all polynomials f of degree less than 100 such that $f(1) + f(2) = 0$;

(**b**) all polynomials f of degree less than 100 such that $f(1) = 0$ and $f(2) = 1$.

(**c**) all polynomials f of degree less than 100 such that $f(1) = f'(1) = 0$;

2. For each subset from the previous question which is a vector space, compute its dimension.

3. (**a**) For $V = \mathbb{R}^3$, show that the vectors

$$f_1 = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}, f_2 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, f_3 = \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}$$

form a basis.

(**b**) Find the coordinates of the vector $\begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix}$ in $\mathbb{R}^3$ relative to the basis f_1, f_2, f_3.

4. Show that the vectors $4 + t$, $2 - t^2$, and $t - t^2$ form a basis of the vector space P_2 of polynomials in t of degree at most 2, and compute the coordinates of the vector $t^2 + 3t$ relative to this basis.

5. A way to fill in the cells of 3×3-matrix with 9 real numbers is called a magic square if sums of numbers in all rows are pairwise equal, and equal to sums of numbers in all columns. Prove that the set of all magic squares is a subspace of $\mathbb{R}^9$, and compute its dimension.