MA 1111: Linear Algebra I
Homework problems due October 22, 2015

Solutions to this problem sheet are to be handed in after our class at 3pm on Thursday. Please attach a cover sheet with a declaration `http://tcd-ie.libguides.com/plagiarism/declaration` confirming that you know and understand College rules on plagiarism.

Please put your name and student number on each of the sheets you are handing in. (Or, ideally, staple them together).

1. For each of the following choices of matrices A and B find out which of the matrices $A+B$, BA, and AB are defined, and compute those which are defined:

(**a**) $A = \begin{pmatrix} 2 & 4 \\ 2 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$;

(**b**) $A = \begin{pmatrix} 2 & 0 & 3 \\ 1 & 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$;

(**c**) $A = \begin{pmatrix} 5 & 1 & 1 \\ 1 & 0 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 4 \\ 0 & 1 \\ 2 & 5 \end{pmatrix}$;

(**d**) $A = \begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 4 \\ 1 & 1 \end{pmatrix}$.

2. Which of the following matrices are invertible? Compute inverses for them. (**a**) $\begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}$; (**b**) $\begin{pmatrix} 6 & 4 \\ 3 & 2 \end{pmatrix}$; (**c**) $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 2 & 0 \end{pmatrix}$; (**d**) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{pmatrix}$.

3. (**a**) Show that if A and B are matrices for which both products AB and BA are defined, then both products AB and BA are square matrices (maybe of different sizes).

(**b**) For an $n \times n$-matrix A, its *trace* $\mathrm{tr}(A)$ is defined as the sum of diagonal elements,

$$\mathrm{tr}(A) = A_{11} + A_{22} + \cdots + A_{nn}.$$

Show that if U is an $n \times m$-matrix, and V is an $m \times n$-matrix, then $\mathrm{tr}(UV) = \mathrm{tr}(VU)$. Explain why this does not contradict the example from class where we found two 2×2-matrices for which $UV \neq VU$.

4. Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. (**a**) Write down explicitly the matrix $A^2 = A \cdot A$. (**b**) Show that if $A^2 = I_2$, then either $A = I_2$, or $A = -I_2$, or $\mathrm{tr}(A) = 0$. (**c**) Give an example of a matrix $A \neq \pm I_2$ for which $A^2 = I_2$.

5. Give an example of a 2×3-matrix A and a 3×2-matrix B for which $AB = I_2$. (*Hint*: in this case, there is already an example with matrices with entries entries from $\{0, 1\}$).