

MA 1111: Linear Algebra I
Homework problems due October 15, 2015

Solutions to this problem sheet are to be handed in after our class at 3pm on Thursday. Please attach a cover sheet with a declaration <http://tcd-ie.libguides.com/plagiarism/declaration> confirming that you know and understand College rules on plagiarism.

Please put your name and student number on each of the sheets you are handing in. (Or, ideally, staple them together).

1. The system of equations

$$\begin{cases} x - y + 3z = -1, \\ 4x - y + z = 2 \end{cases}$$

defines a line ℓ in the 3D space (which is the intersection of the corresponding planes). Find a parametric equation of that line.

2. Using the methods outlined in class, describe the solution set to the system of linear equations

$$\begin{cases} x_1 + x_2 + x_3 + 2x_4 - x_5 = 0, \\ -3x_1 - 4x_2 - 2x_3 + x_5 = -1, \\ 2x_1 + 3x_2 + 2x_3 + 3x_4 + 2x_5 = -2. \end{cases}$$

3. (a) Write down the system of equations corresponding to the 3×5 -matrix

$$A = \begin{pmatrix} 1 & 4 & 5 & 1 & 1 \\ -2 & -2 & -3 & -3 & -4 \\ 1 & -2 & -2 & 2 & -4 \end{pmatrix}$$

(b) Compute the reduced row echelon form of the matrix A , and use it to describe the solution set to the corresponding system of equations.

4. (a) Bring the matrix

$$B = \begin{pmatrix} 1 & 1 & 2 & 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 2 \\ 2 & 1 & 1 & 1 & 1 & 2 & 5 \\ -1 & 2 & 1 & 1 & 0 & 1 & 1 \end{pmatrix}$$

to its reduced row echelon form by elementary row operations.

(b) Which of the unknowns are pivotal (principal) for the corresponding linear system, and which are free? Describe the solution set to that system.

5. Assume that the numbers a , b , c , d , e , and f are such that the system of equations

$$\begin{cases} ax_1 + bx_2 = e, \\ cx_1 + dx_2 = f \end{cases}$$

has (at least) two different solutions. Show that the system

$$\begin{cases} ax_1 + bx_2 = 0, \\ cx_1 + dx_2 = 0 \end{cases}$$

also has (at least) two different solutions. (*Hint*: one solution to this system is $(0, 0)$, so to solve the problem it is enough to find one solution different from $(0, 0)$.)