MA 1111: Linear Algebra I
Homework problems due December 10, 2014

Please put your name and student number on each of the sheets you are handing in. (Or, ideally, staple them together).

1. A sequence $b_0, b_1, \dots$ is defined by $b_0 = 0$, $b_1 = 1$, $b_{n+1} = 3b_n - b_{n-1}$.

(**a**) Show that that $\begin{pmatrix} 0 & 1 \\ -1 & 3 \end{pmatrix}^n \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} b_n \\ b_{n+1} \end{pmatrix}$.

(**b**) Find eigenvalues and eigenvectors of $\begin{pmatrix} 0 & 1 \\ -1 & 3 \end{pmatrix}$ and use them to obtain an explicit formula for b_n.

2. Does there exist a change of basis making the matrix $\begin{pmatrix} 2 & -1 & 1 \\ 0 & 1 & 1 \\ 1 & -1 & 1 \end{pmatrix}$ diagonal? Why?

3. Prove that for every 2×2-matrix A we have

$$A^2 - \operatorname{tr}(A) \cdot A + \det(A) \cdot I_2 = 0.$$

4. Assume that for a 2×2-matrix A we have $A^3 = 0$. Show that in that case we already have $A^2 = 0$.