MA 1111: Linear Algebra I
Selected answers/solutions to the assignment due November 26, 2015

1. We have

$$v = \frac{1}{2}((v+w)-(w-v)),$$
$$w = \frac{1}{2}((v+w)+(w-v)),$$
$$u = (u-2w)+2w = (u-2w)+(v+w)+(w-v),$$

so each of the vectors u, v, and w is a linear combination of those new vectors. Since every vector by assumption is a linear combination of u, v, w, we may substitute the linear combinations obtained to conclude that every vector is a linear combination of the new vectors also.

2. (**a**) Yes. This is the solution set to a homogeneous system of linear equations, and therefore is a subspace as proved in class.

(**b**) No. For example, this subset is not closed under multiplication by real numbers: if $2x-y+z=1$, then $2cx-cy+cz=c\neq 1$ for $c\neq 1$.

(**c**) No. For example, the vector $\begin{pmatrix}1\\1\\0\end{pmatrix}$ belongs to this set, as well as the vector $\begin{pmatrix}0\\-1\\-1\end{pmatrix}$, but their sum, the vector $\begin{pmatrix}1\\0\\-1\end{pmatrix}$ does not belong to U.

3. This system is of the form $Ax=0$, where $A=\begin{pmatrix}2 & -4 & 1 & 1\\ 1 & -2 & 0 & 5\end{pmatrix}$. The reduced row echelon form of the matrix A is $\begin{pmatrix}1 & -2 & 0 & 5\\ 0 & 0 & 1 & -9\end{pmatrix}$, , so x_2 and x_4 are free variables, and the general solution corresponding to the parameters $x_2=s$, $x_4=t$ is $\begin{pmatrix}2s-5t\\ s\\ 9t\\ t\end{pmatrix} = s\begin{pmatrix}2\\1\\0\\0\end{pmatrix}+t\begin{pmatrix}-5\\0\\9\\1\end{pmatrix}$. Therefore we can take $v_1=\begin{pmatrix}2\\1\\0\\0\end{pmatrix}$ and $v_2=\begin{pmatrix}-5\\0\\9\\1\end{pmatrix}$.

4. The reduced row echelon of this matrix is $\begin{pmatrix}1 & 0 & -1 & 1/3\\ 0 & 1 & 0 & -1/3\\ 0 & 0 & 0 & 0\end{pmatrix}$, so x_3 and x_4 are free variables, and the general solution corresponding to the parameters $x_3=s$, $x_4=t$ is $\begin{pmatrix}s-\frac{1}{3}t\\ \frac{1}{3}t\\ s\\ t\end{pmatrix} = s\begin{pmatrix}1\\0\\1\\0\end{pmatrix}+t\begin{pmatrix}-\frac{1}{3}\\ \frac{1}{3}\\ 0\\ 1\end{pmatrix}$. Therefore we can take $v_1=\begin{pmatrix}1\\0\\1\\0\end{pmatrix}$ and $v_2=\begin{pmatrix}-\frac{1}{3}\\ \frac{1}{3}\\ 0\\ 1\end{pmatrix}$.

5. (**a**) We have

$$\mathbf{x}=\mathbf{x}+0=\mathbf{x}+(\mathbf{v}+\mathbf{y})=(\mathbf{x}+\mathbf{v})+\mathbf{y}=0+\mathbf{y}=\mathbf{y}.$$

(**b**) We have

$$\mathbf{v} + (-1) \cdot \mathbf{v} = 1 \cdot \mathbf{v} + (-1) \cdot \mathbf{v} = (1 + (-1)) \cdot \mathbf{v} = 0 \cdot \mathbf{v} = 0,$$

as proved in class. Similarly, $(-1) \cdot \mathbf{v} + \mathbf{v} = 0$. Since the opposite element, as we just proved, is unique, we have $(-1) \cdot \mathbf{v} = -\mathbf{v}$.