MA 1111: Linear Algebra I

Selected answers/solutions to the assignment due October 8, 2015

1. (**a**) No. Otherwise the vectors pointing from the point $(0,1)$ to these points, that is $(5,8) = (5,9)-(0,1)$ and $(8,13) = (8,14)-(0,1)$, would have been proportional, which is not the case. (But they are very close to being proportional, so drawing this on a grid paper might lead to a wrong answer).

(**b**) Yes. Computing the vectors along the sides of that triangle, we get the vectors $(3,4) = (1,1)-(-2,-3)$, $(8,-6) = (9,-5)-(1,1)$, and $(11,-2) = (9,-5)-(-2,-3)$. The scalar product of the first two is $(3,4)\cdot(8,-6) = 24-24 = 0$, so the cosine of the angle between them is 0, and these vectors form a right angle.

2. $(-2,0)$, $(0,-2)$, or $(2,4)$. In general, if $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are given points, then the fourth point is one of $\mathbf{a}+\mathbf{b}-\mathbf{c}$, $\mathbf{b}+\mathbf{c}-\mathbf{a}$, and $\mathbf{c}+\mathbf{a}-\mathbf{b}$. One of possible ideas is to use the parallelogram rule carefully. Another idea: the midpoint of the segment connecting (a_1,a_2) to (b_1,b_2) has coordinates $(\frac{a_1+b_1}{2},\frac{a_2+b_2}{2})$; use the fact that the center of a parallelogram is the midpoint of each of its diagonals.

3. (**a**) No. It is not linear in $\mathbf{v}$, since

$$(\mathbf{u}\cdot(\mathbf{v}_1+\mathbf{v}_2))(\mathbf{v}_1+\mathbf{v}_2)-(\mathbf{u}\cdot\mathbf{v}_1)\mathbf{v}_1-(\mathbf{u}\cdot\mathbf{v}_2)\mathbf{v}_2 = (\mathbf{u}\cdot\mathbf{v}_1)\mathbf{v}_2+(\mathbf{u}\cdot\mathbf{v}_2)\mathbf{v}_1.$$

That latter expression is nonzero for some choices of $\mathbf{u}$, $\mathbf{v}_1$, $\mathbf{v}_2$. For example, if $\mathbf{u}=\mathbf{v}_1=\mathbf{v}_2=\mathbf{i}$, the result is $2\mathbf{i}$.

(**b**) As we established in class, the cross product of two vectors is perpendicular to both of them, so $(\mathbf{u}\times\mathbf{v})\cdot\mathbf{v}=0$ always. As such, it is a multilinear expression: $0+0=0$.

4. We have $|\mathbf{b}|=\sqrt{2^2+1}=\sqrt{5}$, $|\mathbf{c}|=\sqrt{1+3^2}=\sqrt{10}$, and $\mathbf{b}\cdot\mathbf{c}=7$, so we have $\cos\varphi=\frac{7}{\sqrt{50}}$, $\varphi=\cos^{-1}\frac{7}{\sqrt{50}}$.

We also have $|\mathbf{u}|=\sqrt{9}=3$, $|\mathbf{v}|=\sqrt{38}|$, and $\mathbf{u}\cdot\mathbf{v}=18$, so $\cos\varphi=\frac{18}{3\sqrt{38}}=\frac{6}{\sqrt{38}}$.

5. (**a**) This area, as we know, is equal to the length of the vector product of these vectors. We have $\mathbf{u}\times\mathbf{v}=(4,-1,-1)$, so the area is $\sqrt{4^2+1+1}=\sqrt{18}=3\sqrt{2}$. (**b**) This area is the absolute value of $\mathbf{w}\cdot(\mathbf{u}\times\mathbf{v})$, that is $|12-1|=11$.

6. Note that the vectors $\mathbf{v}-\mathbf{u}=(1,1,3)$ and $\mathbf{w}-\mathbf{u}=(2,-2,-1)$ are in this plane, so their cross product is perpendicular to this plane. We have

$$(\mathbf{v}-\mathbf{u})\times(\mathbf{w}-\mathbf{u})=(5,7,-4).$$

This vector is perpendicular to the plane (as is any scalar multiple of it).