1. For the matrix
\[
\begin{pmatrix}
3 & 1 & -1 \\
1 & 1 & -2 \\
-1 & -2 & 1
\end{pmatrix}
\]
of a certain bilinear form, compute the determinants $\Delta_1, \Delta_2, \Delta_3$, and determine the signature of the corresponding quadratic form.

2. Use the Sylvester’s criterion to find all values of the parameter $a$ for which the quadratic form $(18 + a)x_1^2 + 3x_2^2 + ax_3^2 + 10x_1x_2 - (8 + 2a)x_1x_3 - 4x_2x_3$ on $\mathbb{R}^3$ is positive definite.

3. Compute the eigenvalues of the matrix
\[
\begin{pmatrix}
0 & 1 & 0 \\
1 & 0 & 1 \\
0 & 1 & 0
\end{pmatrix}
\]
and determine the signature of the quadratic form
\[
q(x_1e_1 + x_2e_2 + x_3e_3) = x_1x_2 + x_2x_3.
\]

4. Let
\[
\varphi(x_1, x_2) = \sin^2(x_1 - x_2) - e^{\frac{1}{2}(x_1^2 + x_2^2)} + 2cx_1x_2.
\]
Furthermore, let $A$ be the symmetric $2 \times 2$-matrix with entries $a_{ij} = \frac{\partial^2 \varphi}{\partial x_i \partial x_j}(0, 0, 0)$.

(a) Write down the matrix $A$.

(b) Determine all values of the parameter $c$ for which the corresponding quadratic form is positive definite.

(c) Does $\varphi$ have a local minimum at the origin $(0, 0)$ for $c = -3/5$?