MA 1111/1212: Linear Algebra
Tutorial problems, November 20, 2014

1. For each of the following subsets of the set of all polynomials in one variable find out whether this subset is a vector space (with usual operations on polynomials).

(**a**) all polynomials f of degree less than 100 such that $f(1) = 0$;

(**b**) all polynomials f of degree less than 100 such that $f(1) = f(2) = 0$;

(**c**) all polynomials f of degree less than 100 such that $f(1) \cdot f(2) = 0$.

2. For each subset from the previous question which is a vector space, compute its dimension.

3. From previous tutorial, we know that $v_1 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$, $v_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, and $v_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ form a basis of $\mathbb{R}^3$. Compute the coordinates of the vector $\begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix}$ relative to this basis.

4. Show that the vectors $1 + t^2$, $2 - t + t^2$, and $t - t^2$ form a basis of the vector space P_2 of polynomials in t of degree at most 2, and compute the coordinates of the vector $t^2 + 4t + 4$ relative to this basis.