MA 1111/1212: Linear Algebra
Tutorial problems, October 16, 2014

1. Which of the following matrices represent the same permutations? Which of them are even, and which are odd?

(**a**) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 5 & 4 & 3 \end{pmatrix}$; (**b**) $\begin{pmatrix} 1 & 4 & 2 & 3 & 5 \\ 2 & 1 & 5 & 3 & 4 \end{pmatrix}$; (**c**) $\begin{pmatrix} 5 & 3 & 1 & 4 & 2 \\ 3 & 5 & 2 & 4 & 1 \end{pmatrix}$.

2. Describe all values of i, j, k for which the 2×5-matrix

$$\begin{pmatrix} 1 & 4 & 5 & i & 3 \\ 2 & j & k & 5 & 1 \end{pmatrix}$$

represents an odd permutation.

3. Without directly evaluating the determinant, explain why $\det \begin{pmatrix} 4 & 1 & 8 \\ 1 & 5 & 2 \\ 3 & 15 & 6 \end{pmatrix} = 0$.

4. Compute the determinant of the matrix (**a**) $\begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ -1 & 5 & 2 \end{pmatrix}$; (**b**) $\begin{pmatrix} 2 & 1 & -3 & 0 \\ 1 & 5 & 2 & -1 \\ 5 & 0 & 13 & 8 \\ 0 & 1 & 2 & 1 \end{pmatrix}$.

Optional question: Compute the determinant

(**a**) of the matrix $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}$;

(**b**) of the matrix $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 \\ 1 & 2 & 3 & 4 \end{pmatrix}$;

(**c**) of the $n \times n$ matrix A for which $a_{ij} = \min(i, j)$.