MA1S11 (Dotsenko) Tutorial/Exercise Sheet 9

Week 11, Michaelmas 2013

Please hand in your work in the end of the tutorial. Make sure you put your name and student ID number on what you hand in.

A complete solution to each of the questions 1,2 is worth 2 marks, a complete solution to each of the questions 3,4 is worth 3 marks.

Reminder:

- The **antiderivative** and the **indefinite integral**: for a function $f(x)$, the function $F(x)$ is its *anti-derivative* if
$$\frac{dF(x)}{dx} = f(x)$$
The *indefinite integral* is the family of all anti-derivatives
$$\int f(x)\,dx = F(x) + C$$
where C is *the arbitrary constant of integration.*

- **Integration table**

$f(x)$	$\int f(x)\,dx$
x^r $(r \neq -1)$	$\frac{x^{r+1}}{r+1} + C$
e^x	$e^x + C$
$1/x$	$\ln x + C$
$\sin x$	$-\cos x + C$
$\cos x$	$\sin x + C$
$\cos^2 x$	$\tan x + C$

- **Linearity**: if
$$\int f(x)\,dx = F(x) + C, \qquad \int g(x)\,dx = G(x) + C$$
then
$$\int \lambda f(x)\,dx = \lambda F(x) + C \qquad \int [f(x) + g(x)]\,dx = F(x) + G(x) + C$$
where λ is a constant.

- u-**substitution**: If $F'(x) = f(x)$, then the chain rule from the point of view of antiderivatives can be written in the form:
$$\int f(g(x))g'(x)\,dx = F(g(x)) + C.$$
To compute an integral like the LHS one substitutes $u = g(x)$ and $du = \frac{du}{dx}\,dx = g'(x)dx$ and writes
$$\int f(g(x))g'(x)\,dx = \int f(u)\,du = F(u) + C = F(g(x)) + C$$
where in the last step one substitutes back $u = g(x)$ so that x is the independent variable.

- **Integration by parts**: If $F'(x) = f(x)$, and $G'(x) = g(x)$, then the product rule from the point of view of antiderivatives can be written in the form:

$$\int G(x)f(x)\,dx = F(x)G(x) - \int F(x)g(x)\,dx,$$

or equivalently using the notation $du = u'(x)\,dx$ from above

$$\int G\,dF = FG - \int F\,dG.$$

Questions

1. Find the indefinite integrals of $x^5 - 4x$, $\sqrt[3]{x} + 1/\sqrt[3]{x}$, $x^2 - \cos x$ and $(x^3 - 1)(x + 2)$.
2. Evaluate the following integrals using u-substitution:

$$\int (3x^2 - 5)^{1/3} x\,dx, \qquad \int \cos^2(x)\sin(x)\,dx$$

(*Hint:* use substitutions $u = 3x^2 - 5$ and $u = \cos(x)$, respectively.)

3. Evaluate

$$\int \frac{1}{1 - \sin x}\,dx.$$

(*Hint:* first multiply by $1 + \sin x$ above and below the line, then use the identity $\cos^2 x + \sin^2 x = 1$ and u-substitution.)

4. Use (repeated) integration by parts to evaluate the integral $\int (x^3 - 1)e^x\,dx$.