Abstract
In these experiments, some aspects of x-ray absorption spectroscopy were investigated. The x-ray spectrum of molybdenum was recorded and examined at several different electron accelerating voltages. Using this spectrum it was found that \( U\lambda_{\min} \) is constant within error, thus verifying the Duane-Hunt law, and from this Planck’s constant was found to be \( h = 6.3 \pm 0.2 \times 10^{-34} \) Js. The photon energies and wavelengths of the \( K_{\alpha} \) and \( K_{\beta} \) lines were also calculated. X-ray attenuation was then investigated by recording the transmission values of the x-ray beam while thin films of other elements were placed in the beam. From plots of these transmission values, Moseley’s law was verified, and thus Rydberg’s constant was found to be \( R = 1.08 \pm 0.03 \times 10^7 \) m\(^{-1} \), and the K-shell screening parameter was found to be \( \sigma_K = 3.2 \pm 0.6 \). The mass attenuation coefficients, and absorption cross sections of various elements were calculated, and from a plot of the absorption cross-sections versus the atomic numbers for these elements, it was shown that \( \sigma_a \propto Z^4 \).

Introduction
In this experiment, x-rays are produced by firing high energy electrons at a molybdenum target. These x-rays are then diffracted through a crystal based on Bragg’s law, and their spectra observed and recorded in order to complete the aims listed below. Two distinct sets of experiments are carried out:

- **Experiment 1**: This experiment is concerned with the intrinsic properties of both the characteristic spectrum and the continuous spectrum of x-rays.

- **Experiment 2**: This experiment is concerned with the attenuation of x-rays by different materials.

Each of these experiments is outlined in further detail below in the relevant sections, along with the background theory, experimental method, and results and analysis for both.
Experiment 1: The X-ray Spectrum

Introduction

A typical x-ray spectrum produced by an x-ray tube consists of two components. The first of these components is the sharp lines, which are called the characteristic spectrum. These lines are superimposed onto a smooth curve called the continuous spectrum. This second component of the spectrum is due to brehmsstrahlung, radiation produced by the deceleration of the electrons striking the molybdenum target, and it extends to a minimum wavelength $\lambda_{\text{min}}$. The kinetic energy $E$ of the incident electrons is given by

$$E = eU$$  \hspace{1cm} (1)

where $U$ is the accelerating voltage of the electrons (i.e. the tube voltage), and $e$ is the charge on the electron. The energy of the x-ray photon is given by

$$E = h\nu = \frac{hc}{\lambda}$$  \hspace{1cm} (2)

where $h$ is Planck’s constant, $c$ is the speed of light in a vacuum, and $\nu$ and $\lambda$ are the frequency and wavelength of the emitted photon, respectively. Equating equations (1) and (2), we find that the minimum wavelength $\lambda_{\text{min}}$ is given by

$$\frac{hc}{\lambda_{\text{min}}} = eU = E_{\text{max}}$$  \hspace{1cm} (3)

otherwise known as the Duane-Hunt law, or the inverse photoelectric effect.

Incident electrons can knock electrons out of one of the inner shells of the target atom, this gap is then filled by an electron falling from an outer shell, thereby emitting a photon. The characteristic spectrum is due to these electron transitions. We see two distinct peaks, known $E_{K\alpha}$ and $E_{K\beta}$, these are due to electrons transitioning from the L to K shells, and M to K shells respectively. Using the modified Bohr model of the atom, we can find an expression for the binding energy of the electrons in a shell of quantum number $n$

$$E_n = -R\frac{Ze_{\text{eff}}^2}{n^2}$$  \hspace{1cm} (4)

where $Z_{\text{eff}} = Z - \sigma_m$, $R$ is the Rydberg constant, and $\sigma_m$ is the screening constant, which is approximately equal to 1 for the transitions we are concerned with.

In this experiment the x-rays are diffracted by a crystal according to the Bragg law

$$n\lambda = 2d\sin\beta$$

where $n$ is the order of the image (we are only concerned with $n=1$), $d$ is the atomic spacing, and $\beta$ is the angle of diffraction. For an NaCl crystal $2d = 563 \times 10^{-12}$ m, and we are only concerned with the first order image, therefore

$$\lambda = 563 \times 10^{-12} \sin\beta$$  \hspace{1cm} (5)
Aims

The aims of experiment 1 are as follows:

- To record the x-ray spectrum of molybdenum at several different accelerating voltages, $U$, of the electrons.
- To show that $U\lambda_{\text{min}}$ is constant and to determine Planck’s constant.
- To measure the photon energies, and wavelengths, of the $K_{\alpha}$ and $K_{\beta}$ lines in the characteristic spectrum of molybdenum.
- To compare these values with calculated values.

Experimental Method

The x-rays are emitted from a molybdenum anode. Emission is switched off if the leaded glass door of the containment chamber is open. To disperse the x-rays into their spectrum, they are diffracted through a (100) NaCl crystal, using Bragg geometry. The crystal is clamped against a small horizontal bar in its preferred orientation. The spectra of $I$ vs $\beta$ is recorded for various tube voltages. From the spectra, the $\lambda_{\text{min}}$ values and the wavelengths of the $K_{\alpha}$ and $K_{\beta}$ lines are determined. The corresponding photon energies, $E_{\text{max}}$, $E_{K_{\alpha}}$, and $E_{K_{\beta}}$ are calculated. The values of these quantities are tabulated and it is verified that the $U\lambda_{\text{min}}$ values are constant within error. Planck’s constant is calculated. Using the modified Bohr model, the expected values of $E_{K_{\alpha}}$ and $E_{K_{\beta}}$ for molybdenum are calculated and compared with the measured values.

Results and Analysis

The x-ray spectra obtained in this experiment are shown below in figure 1. From the spectra, the location of $\beta_{\text{min}}$ was estimated for each $U$, then using equation (5) the values for $\lambda_{\text{min}}$ were calculated.

![Figure 1: X-ray spectra of molybdenum for tube voltages of 35, 30, 25, and 20 kV](image)
The values of the product $U\lambda_{\text{min}}$ were then calculated and tabulated below in table 1. It is clear that the value of $U\lambda_{\text{min}}$ is constant within experimental error. To calculate the maximum photon energies $E_{\text{max}}$ for each tube voltage, equation (3) was used. This equation was also used to calculate Planck’s constant for each $U$.

<table>
<thead>
<tr>
<th>$U$ (V)</th>
<th>$\lambda_{\text{min}}$ (pm)</th>
<th>$U\lambda_{\text{min}}$ ($10^{-6}$ V m)</th>
<th>$E_{\text{max}}$ (keV)</th>
<th>$h$ ($10^{-15}$ eVs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>35000 ± 100</td>
<td>33 ± 1</td>
<td>1.17 ± 0.05</td>
<td>37 ± 1</td>
<td>3.9 ± 0.2</td>
</tr>
<tr>
<td>30000 ± 100</td>
<td>39 ± 1</td>
<td>1.18 ± 0.04</td>
<td>32 ± 1</td>
<td>3.9 ± 0.2</td>
</tr>
<tr>
<td>25000 ± 100</td>
<td>48 ± 1</td>
<td>1.20 ± 0.04</td>
<td>25.8 ± 0.8</td>
<td>4.0 ± 0.1</td>
</tr>
<tr>
<td>20000 ± 100</td>
<td>61 ± 1</td>
<td>1.22 ± 0.03</td>
<td>20.4 ± 0.5</td>
<td>4.1 ± 0.1</td>
</tr>
</tbody>
</table>

Table 1: Experimental values for $U$ and $\lambda_{\text{min}}$

Taking the average of the values for Planck’s constant we obtain a value of $h = 3.9 ± 0.1 \times 10^{-15}$ eVs. This is equivalent to $h = 6.3 ± 0.2 \times 10^{-34}$ Js.

The wavelengths of the $K_\alpha$ and $K_\beta$ lines were found in a similar manner to $\lambda_{\text{min}}$. To find the corresponding photon energies $E_{K_\alpha}$ and $E_{K_\beta}$, equation (2) was used. The values calculated for each of these quantities are tabulated below in table 2.

<table>
<thead>
<tr>
<th>$\lambda_{K_\alpha}$ (pm)</th>
<th>$\lambda_{K_\beta}$ (pm)</th>
<th>$E_{K_\alpha}$ (keV)</th>
<th>$E_{K_\beta}$ (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>70.6 ± 0.9</td>
<td>62.8 ± 0.9</td>
<td>17.5 ± 0.3</td>
<td>19.8 ± 0.3</td>
</tr>
</tbody>
</table>

Table 2: Experimental values for photon energies and wavelengths of the $K_\alpha$ and $K_\beta$ lines

For $E_{K_\alpha}$ we are concerned with the transition from the L to K shell, so we take $E_{K_\alpha} = E_2 - E_1$. For $E_{K_\beta}$ we are concerned with the transition from the M to K shell, so we take $E_{K_\beta} = E_3 - E_1$. We then derive the following results

$$E_{K_\alpha} = \frac{3Rhc}{4}Z_{\text{eff}}^2, \quad E_{K_\beta} = \frac{8Rhc}{9}Z_{\text{eff}}^2$$

Taking $\sigma_m \approx 1$, we find that the expected values are $E_{K_\alpha} \approx 17.584$ keV and $E_{K_\beta} \approx 19.77$ keV.
Experiment 2: X-ray Attenuation

Introduction

When x-rays pass through a material their intensity is reduced as a result of both absorption and scattering according to the following relationship

\[ T = \frac{I}{I_0} = \frac{R}{R_0} \quad (6) \]

where \( T \) is the transmittance and \( I, I_0, R, \) and \( R_0 \) are the intensities and count rates with and without the foils respectively. The decrease in absorption is proportional to the atomic number of the absorbing element, and inversely proportional to the incident x-ray photon energy. At certain energies there is a an abrupt change in absorption, these are called the absorption edges.

For a slab of material of thickness \( x \) the following relationship holds

\[ I = I_0 e^{-\sigma n x} = I_0 e^{-\mu x} \quad (7) \]

where \( I_0 \) is the incidence intensity, \( \mu = \sigma n \) is the linear attenuation coefficient, and \( \sigma \) is the removal cross section.

From these equations (4) and (5) we can then see that

\[ \mu = \frac{-\ln T}{x} \quad (8) \]

Using \( n = \frac{N_a \rho}{A} \) where \( N_a \) is Avogadro’s number, and \( A \) is the atomic weight of the material of density \( \rho \), then

\[ \sigma = \frac{\mu}{n} = \frac{A \mu}{N_a \rho} \quad (9) \]

where \( \mu/\rho \) is called the mass attenuation coefficient.

Since photons can be removed from the beam by either absorption or scattering, \( \sigma \) can be written as

\[ \sigma = \sigma_a + \sigma_s \]

where \( \sigma_a \) and \( \sigma_s \) are the partial removal cross-sections associated with absorption and scattering, respectively. In the case of this experiment \( \sigma_s < \sigma_a \) and can be approximated as

\[ \sigma_s \approx 0.02A \frac{1}{N_a} \]

We can therefore find \( \sigma_a \) for each foil

\[ \sigma_a \approx \left[ \frac{\mu}{\rho} - 0.02 \right] A \frac{1}{N_a} \quad (10) \]

Absorption of energy can take place via various mechanisms. In these experiments, the x-ray photons have energies in the range of 10 - 40 keV, and so attenuation is dominated by the photoelectric effect, i.e. the energy of the photons is used to eject a bound electron from the absorbing material. For the ejection of a K shell electron, we require

\[ h\nu \geq \text{binding energy of K shell electron} \]

A jump in attenuation is seen when the incident photon energy reaches this binding energy, for the K shell this jump is known as a K-absorption edge. At this edge

\[ E_K = \frac{hc}{\lambda_K} = Rhc(Z - \sigma_K)^2 \quad (11) \]
where $E_K$ is the energy at absorption edge, i.e the wavelength of the absorption edge $\lambda_K$ is given by Moseley’s law

$$\sqrt{RZ} = \sqrt{\frac{1}{\lambda_K}} + \sqrt{R\sigma_K}$$

(12)

At a fixed wavelength, away from an edge, the absorption removal cross-section $\sigma_a$ increases rapidly as the atomic number $Z$ increases, i.e $\sigma_a \propto Z^p$, otherwise written as

$$\sigma_a = CZ^p$$

where C is a constant. Taking the natural log of both sides we find that

$$\ln \sigma_a = p \ln Z + \ln C$$

(13)

Section A: K-absorption Edge

Aims

The aims of experiment 2, section A are as follows;

- To determine the energy, $E_K$ at the K absorption edge for zirconium, molybdenum, silver and indium.
- To show that $E_K \propto (Z - \sigma_K)^2$.
- To determine the Rydberg constant, R, and the value of $\sigma_K$.

Experimental Method

The experimental apparatus is the same as that used in experiment 1. As before, the NaCl crystal is used to disperse the spectrum. To determine the K absorption edge for several different materials, the x-ray spectrum of molybdenum is recorded with and without thin foils of these metals inserted in the x-ray beam. All 5 spectra are printed on a single plot. Transmission values are calculated for a couple of the data points and compared with the plotted values. From the plots of $T$ versus $\lambda$, $\lambda_k$ and $E_k$ in keV for each foil are evaluated and tabulated. A suitable graph to verify the relation $E_k = R hc(Z - \sigma_K)^2$ is plotted, and from this graph both the Rydberg constant R and the K shell screening parameter $\sigma_k$ are determined.
Results and Analysis

The spectra obtained in this experiment are shown below in figure 2

Figure 2: X-ray spectra for molybdenum with foils of Zr, Mo, Ag, and In placed in the beam
A plot of the transmission of each foil versus $\lambda$ was also obtained, shown below in figure 3.

![Transmission plot](image)

**Figure 3:** Transmission of x-rays of various wavelengths through foils of Zr, Mo, Ag, and In

In order to ensure the accuracy of the values produced by the computer, transmission values were calculated for some of the data points, using equation (6) these are tabulated below in table 3.

<table>
<thead>
<tr>
<th>Value</th>
<th>Zr</th>
<th>Mo</th>
<th>Ag</th>
<th>In</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculated</td>
<td>0.85556</td>
<td>0.2556</td>
<td>0.3673</td>
<td>0.0466</td>
</tr>
<tr>
<td>Plotted</td>
<td>0.856</td>
<td>0.256</td>
<td>0.367</td>
<td>0.047</td>
</tr>
</tbody>
</table>

**Table 3:** Calculated and plotted Transmission values

From this plot the wavelength of the K-absorption edge, $\lambda_K$, for each foil was estimated, and using equation (3), the energy at the K-absorption edge, $E_K$ was calculated. These values are tabulated below in table 4.

<table>
<thead>
<tr>
<th>Element</th>
<th>Atomic Number</th>
<th>$\lambda_K$ (pm)</th>
<th>$E_K$ (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zr</td>
<td>40</td>
<td>68 ± 2</td>
<td>18.2 ± 0.6</td>
</tr>
<tr>
<td>Ag</td>
<td>42</td>
<td>62 ± 2</td>
<td>20.1 ± 0.6</td>
</tr>
<tr>
<td>Mo</td>
<td>47</td>
<td>48 ± 2</td>
<td>26 ± 1</td>
</tr>
<tr>
<td>In</td>
<td>49</td>
<td>44.3 ± 0.9</td>
<td>28.1 ± 0.6</td>
</tr>
</tbody>
</table>

**Table 4:** Wavelengths and photon energies of the K-absorption edge
A graph of $\lambda_{K}^{-1/2}$ versus the atomic number, $Z$, was then plotted, seen below in figure 4 and using equation (12), values for the Rydberg constant, $R = 1.08 \pm 0.03 \times 10^7 \text{m}^{-1}$ and the K shell screening parameter $\sigma_K = 3.2 \pm 0.6$ were found.

![Graph](image)

(a) Plot showing intercept  (b) Zoom of the region of interest

**Figure 4: Plot of K-edge wavelength versus atomic number**

**Section B: Mass Attenuation Coefficient and Absorption Cross-Section**

**Aims**

The aims of experiment 2, section B are as follows;

- To determine the mass attenuation coefficient, $\mu/\rho$, and absorption cross-section, $\sigma_a$, for aluminium, iron, copper, and zirconium, at a fixed wavelength away from the absorption edges.

- To find the dependence of $\sigma_a$ on $Z$ at this wavelength.

**Experimental Method**

The experimental apparatus is the same as that used in the previous experiments. As before, the NaCl crystal is used to disperse the spectrum. Both $\mu/\rho$ and $\sigma_a$ are calculated for each foil by measuring its transmittance at a chosen wavelength, well below an edge but not so low that the intensity has become too small, the value chosen for this experiment was 41.3pm. The count rate is measured first with no foil, and then with each of the foils respectively, thus the transmittance for each foil is calculated.

A graph of $\sigma_a$ versus $Z$ is plotted, then a suitable graph to show that $\sigma_a \propto Z^p$ is plotted and $p$ is determined.
Results and Analysis

The count rates for each foil were compared to a base count rate with no foil to calculate transmission values according to equation (6). Values for $\mu$ were then calculated using equation (8), and using the given density values, the mass attenuation coefficient, $\mu/\rho$, was then calculated for each foil. The absorption cross sections for each foil, $\sigma_a$, were then found using equation (10). These values are tabulated below in table 5.

<table>
<thead>
<tr>
<th>Element</th>
<th>Atomic Number</th>
<th>$\mu/\rho \times 10^{-2} (\text{m}^2\text{kg}^{-1})$</th>
<th>$\sigma_a \times 10^{-25} (\text{m}^2) (=\text{kb})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al</td>
<td>13</td>
<td>$9.7 \pm 0.7$</td>
<td>$35 \pm 2$</td>
</tr>
<tr>
<td>Fe</td>
<td>26</td>
<td>$79 \pm 5$</td>
<td>$721 \pm 51$</td>
</tr>
<tr>
<td>Cu</td>
<td>29</td>
<td>$91 \pm 6$</td>
<td>$944 \pm 67$</td>
</tr>
<tr>
<td>Zr</td>
<td>40</td>
<td>$237 \pm 17$</td>
<td>$3579 \pm 253$</td>
</tr>
</tbody>
</table>

Table 5: Experimental values for the mass attenuation coefficients and absorption cross-sections

A graph of $\sigma_a$ versus $Z$ was then plotted, see figure 5a, and also a plot of $\ln \sigma_a$ versus $\ln Z$ to show determine the constant $p$ as seen in equation (13), see figure 5b.

![Graph](image)

Figure 5: Plots of absorption cross-section versus atomic number
Conclusions

Overall the experiment was quite successful, the values obtained throughout were consistent with accepted values. In experiment 1, it was shown that $U\lambda_{\text{min}}$ is constant within experimental value, and from these values Planck’s constant was calculated to be $h = 6.3 \pm 0.2 \times 10^{-34}$ J s, which is within 1% of the accepted value of $h = 6.626 \times 10^{-34}$ J s, any discrepancy can be explained by the difficulty in judging the position of $\lambda_{\text{min}}$. Using the values of $\lambda_{K\alpha} = 70.6 \pm 0.9$ m and $\lambda_{K\beta} = 62.7 \pm 0.9$ m from the spectrum, values for the corresponding photon energies $E_{K\alpha} = 17.6 \pm 0.3$ keV and $E_{K\beta} = 19.8 \pm 0.3$ keV were found, which agree with their expected values.

In experiment 2 it was verified that the computer software was plotting sufficiently accurate data points for the transmission of each foil. From a plot of $\lambda_K$ versus the atomic number $Z$, Moseley’s law was verified, and from this plot the Rydberg constant, and the K-shell screening parameter were evaluated as $R = 1.08 \pm 0.03 \times 10^7$ m$^{-1}$ and $\sigma_K = 3.2 \pm 0.6$, respectively. Using the transmission values, the values of $\mu/\rho$ and $\sigma_a$ were calculated for Al, Fe, Cu and Zr at a wavelength of 41 pm. These values agreed very well with accepted experimental values. From a logarithmic plot of absorption cross-section versus atomic number, it was found that $p = 4$, and therefore $\sigma_a \propto Z^4$. 