The Cornu Method

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Abstract:

The aims of this experiment were to obtain values for Young's modulus and Poisson's ratio for Perspex using the Cornu Method. A value of $2.63\text{GPa} \pm 0.638\text{GPa}$ was found for Young's modulus, which is in the accepted range, within experimental error. However, Poisson's ratio was found to be $0.54\pm0.02$, which is outside the acceptable values, therefore the experiment was only a partial success. The experiment was hampered by the difficulty in obtaining accurate results, due to the subjective nature of the fringes, and difficulty in reading the Vernier scale.

Aims:

Our aims in this experiment were;

- To determine Young's modulus for Perspex using an interference method
- To determine Poisson's ratio also for Perspex using the same method
- To observe the change in interference pattern when a convex lens is used in place of an angled slide.

Introduction and Theory:

Young's modulus ($Y$) of a material is the ratio of the stress on that material to the strain of it. It is essential a measure of the stiffness of a material, or how easy it is to deform that material.

$$Y = \frac{\text{stress}}{\text{strain}}$$

Stress of a material is the force applied to it per unit area, which causes a deformation in the material.

$$\text{Stress} = \frac{\text{Force}}{\text{Area}}$$

Strain of a material is the ratio of the change in length to the original length, i.e. how much of a deformation is created.

$$\text{Strain} = \frac{\Delta l}{l_0}$$

In Fig. 1 below, we see a Perspex beam with a mass (m) suspended from either end. These masses cause the beam to bend in both the longitudinal and transverse directions. This creates an internal bending moment which opposes that produced by the load, which is given by

$$\frac{YAk^2}{R_1}$$
Where \( Y \) is Young’s modulus, \( A \) is the cross sectional area of the beam, \( R_1 \) is the longitudinal radius of curvature, and \( k \) is the radius of gyration, given by \( k = \frac{b}{\sqrt{12}} \) for our beam, where \( b \) is the thickness of the beam.

The beam is then, in equilibrium when the forces opposing the internal stresses balance out the forces caused by the load, i.e.

\[
mgl = \frac{YAk^2}{R_1}
\]

Or, by rearranging, Young’s modulus is given by;

\[
Y = \frac{mglR_1}{Ak^2}
\]

Poisson’s ratio, \( \sigma \), is defined as the ratio of lateral strain to longitudinal strain,

\[
\sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{R_1}{R_2}
\]

Where \( R_1 \) is the longitudinal radius of curvature and \( R_2 \) is the transverse radius of curvature. It is a fact that when a material is stretched in the lateral direction, it will compress in the longitudinal direction, and vice versa, and Poisson’s ratio measures this tendency.

![Diagram](image)

When the beam is bent, a longitudinal stress will cause a lateral strain. The loci of points of constant vertical displacement, \( d \), under this stress are given by

\[
\frac{x^2}{R_1} - \frac{y^2}{R_2} = 2(d - d_0)
\]

This equation describes a hyperbola, which implies that the fringes we obtain form two sets of hyperbolae with common asymptotes given by \( x^2R_2 = y^2R_1 \). Taking \( \theta \) as the angle between the asymptotes and the x-axis we find that
When only considering the x-axis, we have $y=0$, and the first equation above becomes

$$x^2 = 2R_1 (d - d_0)$$

But from optical theory, we know that for constructive interference, i.e. on a fringe, $2d = N\lambda$ where $N$ is an integer, and so we get

$$x^2 = R_1 (N\lambda - 2d_0)$$

And similarly, along the y-axis we get

$$y^2 = -R_2 (N\lambda - 2d_0)$$

Experimental Method:

- The experimental apparatus is shown in Fig. 1. Measurements were taken for $l$, $b$ and the width of the beam, $w$. The area, $A$, was then calculated.
- A glass slide angled at 45° was placed on the glass plate beneath the travelling microscope. A sodium lamp was shone upon it the angled slide such that the glass slide was illuminated by light reflected from the sodium lamp.
- The microscope was centred at the hyperbola and the fringes located in both directions. The distances between fringes was first measured in the x-direction. By rotating the bar by 90°, the distance between fringes was then measured in the y-direction.
- By observing the asymptotes using a lens, the angle $\theta$ was estimated using a protractor.
- The 45° glass slide was then replaced with a convex lens and the new pattern observed was recorded.

Results and Analysis:

The following measurements were taken; $l = 0.136m \pm 0.001m$, $b = 0.006m \pm 0.001m$, and $w = 0.040m \pm 0.001m$. Also the wavelength of sodium light, $\lambda = 589.3 \times 10^{-9} m$.

![Figure 2](image-url)
The slope of the line in Fig. 2 is $m_x = 8.37 \times 10^{-7} \pm 3.58 \times 10^{-8}$. From the equation obtained above for $x^2$, the value for $R_1$ is given by:

$$R_1 = \frac{m_x}{A} = 1.42 \pm 0.06$$

The slope of the line in Fig. 3 is $m_y = 1.54 \times 10^{-6} \pm 1.65 \times 10^{-8}$. From the equation obtained above for $y^2$, the value for $R_2$ is given by:

$$R_2 = \frac{m_y}{A} = 2.61 \pm 0.03$$

Using the values of $b$ and $w$, the value for $A$ was found to be $A = 2.4 \times 10^{-4} m^2 \pm 0.4 \times 10^{-4} m^2$.

From the expression above for Young’s Modulus, the value of $Y$ was calculated to be

$$Y = \frac{mglR_1}{Ak^2} = \frac{12mglR_1}{Ab^2} = 2.63 GPa \pm 0.638 GPa$$

Where the error in $Y$ is calculated as follows;

$$\Delta Y = Y \sqrt{\left(\frac{\Delta l}{l}\right)^2 + \left(\frac{\Delta R_1}{R_1}\right)^2 + \left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta b}{b}\right)^2} = 0.63 GPa$$

The value of $\theta$, the angle between the asymptotes and the x-axis was estimated to be 60°. From the formula for Poisson’s ratio found above, a value for $\sigma$ was estimated to be

$$\cot^2 \theta = \sigma = 0.333$$

And from the other formula for Poisson’s ratio, $\sigma$ was calculated as

$$\frac{R_1}{R_2} = \sigma = 0.54 \pm 0.02$$
Finally, when the angled glass slide was replaced with a convex lens, Newton’s Rings were observed, (as seen in Fig. 4) due to light interfering on the surface of the beam and the bottom of the lens, caused by the light travelling through two media of different refractive indices, namely glass and air.

![Fig. 4 Newton’s Rings.](image)

**Discussions and Conclusions:**

- Young’s Modulus for a Perspex beam was found to be $2.63 \pm 0.638 \text{GPa}$. This is within the accepted range of 2.7GPa-3.5GPa, and this part of the experiment was quite successful.
- The value estimated for Poisson’s ratio was 0.333, and when calculated we obtained a value of 0.54±0.02. Our values for the estimate and the calculated ratio did not agree well, our estimate being too low to be in the range, and our calculated value too high. We put this down to the many errors involved in this part of the experiment.
- During the course of this experiment, it was obvious that a great many errors were being introduced, from the subjective nature of each fringe, to the errors arising from the reading of the Vernier scale. The failure of the second half of the experiment may be due to the difficulty in eradicating, or at least accounting for these errors.